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Semiconductor Measurement Technology:

TXYZ: A Program for Semiconductor IC Thermal Analysis

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Semiconductor Measurement Technology:

TXYZ: A Program for Semiconductor IC Thermal Analysis

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Semiconductor Measurement Technology:

TXYZ: A Program For Semiconductor IC Thermal Analysis

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Abstract

A computer program, TXYZ, for the thermal analysis of semiconductor integrated circuits is presented and its applications are discussed. The program makes use of the closed form, analytic solution of the steady-state heat flow problem for a rectangular three-layer structure with multiple heat sources on the top layer. The temperature may be obtained for any point or set of points in the structure and is useful in the determination of the steady-state thermal response of IC chips and packages.

Key words: FORTRAN; Fourier analysis; integrated circuit; semiconductor; steady-state heat flow; thermal analysis; thermal resistance.

INTRODUCTION

Since the introduction of semiconductor integrated circuits, one of the most important sources of device failure is the lack of temperature control. Hence, an accurate physical picture of the temperature distribution in the device package under the power condition of actual operation is of utmost importance. The purpose of this report is to describe the program, TXYZ, which has been developed for the thermal analysis of integrated circuit packages. In particular, the basic physical model and the mathematical analysis are described and an annotated listing of the program, along with sample data, is presented.

The physical and mathematical model used here is taken in part from work previously carried out by Kokkas [1]. In order to provide the reader with a self-contained document for using TXYZ, much of the development presented by Kokkas has been worked out and specialized to the steady-state heat flow problem. The discussion which follows presents an annotated description of Kokkas' analysis with additional material added where necessary. In particular, the equations have been analyzed in detail so as to investigate the convergence of the solutions used in the numerical implementation. Specifically, the form of the solutions for small and large values of the argument is shown to require special consideration to avoid numerical overflow problems.

This report is naturally broken up into two parts. The first deals with the mathematical and numerical details of the construction of the program. The second deals with the program and its use. For those readers who are interested in the use of the program, the section entitled "GENERAL DISCUSSION OF THE TXYZ

PROGRAM" begins the portion of the report where the program and specific examples are discussed.

UNITS

The American semiconductor industry has traditionally used mixed English and metric units, but presently there is a trend in the direction of the International System (SI) units. For the purpose of conversion, it should be noted that 1 mil = 0.001 in. = 25.4 μm and that 1 cal/s = 4.184 W.

SOLUTION OF STEADY-STATE HEAT FLOW EQUATION:

SINGLE RECTANGULAR LAYER

Consider a material of uniform thermal conductivity (κ_1), in the form of a rectangular box of lateral dimensions L_x, L_y and thickness L_1. The problem is to determine the temperature, $T(x, y, z)$, inside the material. It is assumed that the temperature satisfies the steady-state heat flow equation [2]

$$\nabla^2 T(x, y, z) = 0. \tag{1}$$

As this equation is second order in the three coordinates, there are six boundary conditions. Four of these will be provided by the lateral boundary conditions. In the present problem, all four of the lateral boundary conditions are provided by the assumption that there is no heat flow out of the lateral boundaries of the material, i.e.,

$$\frac{\partial T(x, y, z)}{\partial x}\Big|_{x=0,L_x} = \frac{\partial T(x, y, z)}{\partial y}\Big|_{y=0,L_y} = 0. \tag{2}$$

The remaining two boundary conditions will be provided by the vertical boundary conditions (in z). These vertical boundary conditions will not be specified at the present time as the intent of the present section is to obtain a general solution of the one-layer problem with only the lateral boundary conditions being specified. As the above equation is formulated in Cartesian coordinates, it is convenient to use Fourier analysis techniques to solve the x and y portion of the equation. The Fourier transform with respect to the variables x and y is used, remembering that the geometry is constrained to $0, L_x$ and $0, L_y$. This is defined as [3]

$$\tau(f_x, f_y, z) = \int_0^{L_x} \int_0^{L_y} T(x, y, z) \exp(-2\pi i(x f_x + y f_y)) dx dy, \tag{3}$$

where f_x, f_y are the Fourier transform variables which are conjugate to the variables x, y. The inverse Fourier transform is defined as

$$T(x, y, z) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \tau(f_x, f_y, z) \exp(2\pi i(x f_x + y f_y)) df_x df_y. \tag{4}$$

The requirement that there is no heat flow out of the sides of the structure, i.e., $\partial T(x, y, z)/\partial x$ and $\partial T(x, y, z)/\partial y$ are zero when x and y are either equal to zero or to L_x or L_y, respectively, leads to a consideration of the expression

$$\frac{\partial T(x, y, z)}{\partial x} =$$

$$\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \tau(f_x, f_y, z) \exp(2\pi i y f_y) 2\pi f_x \Big\{ -\sin(2\pi f_x x) + i \cos(2\pi f_x x) \Big\} df_x df_y, \tag{5}$$

and a similar expression for $\partial T(x, y, z)/\partial y$. If this is to be zero at the origin, this would require that the cos term be removed. Next, consider the resulting expression at the other lateral boundary, i.e., at $x = L_x$ where it is supposed to be zero. The only way that this could be the case is if the argument of the sin function is an integer times π, or that the Fourier transform variable is of the form

$$f_x = \frac{n}{2L_x}. \tag{6}$$

The same argument applies to the y-dependent portion. Then, in the Fourier representation, the temperature (with the above lateral boundary conditions) may be written as

$$T(x, y, z) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \tau(n, m, z) \cos(n\pi x/L_x) \cos(m\pi y/L_y) df_x df_y. \tag{7}$$

The Fourier transform equation is now written as

$$\tau(n,m,z) = \int_0^{L_x}\int_0^{L_y} T(x,y,z)\cos(n\pi x/L_x)\cos(m\pi y/L_y)dxdy. \tag{8}$$

As the system is of finite size, it is convenient to write the integral in eq (7) as a sum over the Fourier cos terms which fit into the rectangular geometry. Further, as the cos function is symmetric around the origin, the sums may be written over only the non-negative values of the indices. It is important to keep in mind that the terms corresponding to $m, n{=}0$ do not have the factor of 2 coming from the symmetry of the cos function. In addition, the differentials may be written as

$$df_x = \Delta\frac{n}{L_x} = \frac{n+1}{L_x} - \frac{n}{L_x} = \frac{1}{L_x}. \tag{9}$$

Then, the Fourier representation of the temperature may be written as

$$T(x,y,z) = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \frac{4\tau(n,m,z)\cos(n\pi x/L_x)\cos(m\pi y/L_y)}{(\delta_{n0}+1)(\delta_{m0}+1)L_xL_y}, \tag{10}$$

where $\delta_{nn'}$ is the Kronecker delta and is equal to unity if $n = n'$ and zero otherwise. By substituting eq (10) into eq (1) and using

$$\nabla^2 = \left\{\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right\}, \tag{11}$$

and

$$\frac{\partial^2}{\partial x^2}\cos(n\pi x/L_x) = -(n\pi/L_x)^2\cos(n\pi x/L_x), \tag{12}$$

and the same relation for the y-dependence, it is straightforward to show that

$$\sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \frac{4\cos(n\pi x/L_x)\cos(m\pi y/L_y)}{(\delta_n+1)(\delta_m+1)L_xL_y}\left\{-(n\pi/L_x)^2-(m\pi/L_y)^2+\frac{\partial^2}{\partial z^2}\right\}\tau(n,m,z) = 0. \tag{13}$$

As the sum is zero for arbitrary values of the variables x and y (and the cos terms are, in general, nonzero), then a necessary and sufficient condition that eq (13) is satisfied is that

$$\left\{-(n\pi/L_x)^2-(m\pi/L_y)^2+\frac{\partial^2}{\partial z^2}\right\}\tau(n,m,z)=0. \tag{14}$$

This differential equation may be solved analytically using elementary methods. If the variable, γ, is defined as

$$\gamma=\left\{\left(\frac{n\pi}{L_x}\right)^2+\left(\frac{m\pi}{L_y}\right)^2\right\}^{1/2}, \tag{15}$$

then the above equation may be written in the form

$$\frac{\partial^2}{\partial z^2}\tau(n,m,z)-\gamma^2\tau(n,m,z)=0. \tag{16}$$

The solution of this equation is

$$\tau(n,m,z)=\alpha\cosh(\gamma z)+\beta\sinh(\gamma z), \tag{17}$$

where the coefficients α and β, which may be functions of γ, are determined from the two z-dependent boundary conditions. The above equation is the general solution for the z-dependent Fourier expansion coefficients for a single rectangular layer. In the discussion of the problem of a rectangular j-layer structure where all of the layers have the same lateral dimensions, the solution in each of the layers can be expressed in the form of the above equation where the coefficients are to be determined from the two z-dependent boundary conditions appropriate to each of the layers. This will be used in the next section where the three-layer problem will be discussed.

SOLUTION OF STEADY-STATE HEAT FLOW EQUATION:

THREE-LAYER RECTANGULAR STRUCTURE

The basic problem considered in this section is the calculation of the three-dimensional temperature distribution in a three-layer structure which is assumed to be of the geometric form presented in figure 1. The three layers are characterized in terms of the thermal conductivities and thicknesses $\kappa_i, L_i (i = 1, 2, 3)$. One particular example of a three-layer structure is an IC package. For this particular case, the top layer is the semiconductor device, whereas the middle and bottom layers are the die attach and substrate layers. Clearly, the thicknesses and thermal conductivities of these three layers are of considerable importance in the dissipation of heat generated by the power sources on the surface of the semiconductor device. These power sources are typically the regions at or near the surface of the device where currents are passed into the device during normal operation. Consequently, the generation of heat in the device is one of the unavoidable side effects of device operation.

All three layers of the model are assumed to have the same lateral dimensions. This assumption greatly simplifies the analysis in terms of Fourier series which are common to all three layers. In addition, it is assumed that there is no heat flow out of the lateral boundaries of the structure due to either convection or radiation. Again, this makes the analysis more manageable.

The mathematical formulation of this problem is based upon the following set of assumptions:

1) the lateral dimensions of the layers in the structure are all equal while the

thicknesses may be different,

2) there is no heat loss from the lateral surfaces due to either radiation or convection and heat flow in the structure takes place by conduction,

3) there are no heat losses due to interconnections to the chip (the top layer),

4) there is no thermal contact resistance between the various layers which are in contact,

5) the heat sink, which is in contact with the bottom layer, is ideal and has a temperature equal to ambient,

6) each layer is of uniform, isotropic, temperature-independent thermal conductivity, and

7) there is no input power density inside the structure, heat is generated only on the top surface.

The starting equation for the analysis is the steady-state homogeneous heat flow equation as discussed for the single-layer problem,

$$\nabla^2 T(x, y, z) = 0. \tag{18}$$

In the following discussion, the temperature in the first layer will be referred to as $T_1(x, y, z)$, while the temperature in the second and third layers will be denoted by $T_2(x, y, z)$ and $T_3(x, y, z)$, respectively. Built into the above assumptions is the fact that the temperature and its normal derivative (proportional to the normal heat flow) are continuous at the interfaces between the layers. The assumptions that heat

enters the structure only on the top surface through the heating elements and that the bottom of the bottom layer is at the same temperature as the heat sink are also available. These provide the six z-dependent boundary conditions on the system of equations generated by using the solution of the homogeneous single-layer problem. These boundary conditions may be expressed as follows. First, the assumption that there is no heat flow out of the lateral boundaries is written as

$$\frac{\partial T_i(x,y,z)}{\partial x}\Big|_{x=0,L_x} = \frac{\partial T_i(x,y,z)}{\partial y}\Big|_{y=0,L_y} = 0, (i = 1,2,3). \tag{19}$$

Next, the assumption that heat enters only where the power is applied in the first layer is expressed as

$$\kappa_1 \frac{\partial T_1(x,y,z)}{\partial z}\Big|_{z=0} = P(x,y), \tag{20}$$

where κ_1 is the thermal conductivity of the top layer. Further, the assumption that the temperature is continuous across the interfaces between the layers may be written as

$$T_1(x,y,z)|_{z=-L_1} = T_2(x,y,z)|_{z=-L_1}, \tag{21}$$

$$T_2(x,y,z)|_{z=-(L_1+L_2)} = T_3(x,y,z)|_{z=-(L_1+L_2)}. \tag{22}$$

The assumption that the heat flow is continuous across the interfaces between the layers is expressed by the conditions that

$$\kappa_1 \frac{\partial T_1(x,y,z)}{\partial z}\Big|_{z=-L_1} = \kappa_2 \frac{\partial T_2(x,y,z)}{\partial z}\Big|_{z=-L_1}, \tag{23}$$

$$\kappa_2 \frac{\partial T_2(x,y,z)}{\partial z}\Big|_{z=-(L_1+L_2)} = \kappa_3 \frac{\partial T_3(x,y,z)}{\partial z}\Big|_{z=-(L_1+L_2)}. \tag{24}$$

Finally, the assumption that the temperature is continuous across the interface between the third layer and the heat sink is written as

$$T_3(x, y, z)|_{z=-L_z} = T_a = 0, \tag{25}$$

where all temperatures are measured relative to the ambient heat sink temperature. In the above equations, κ_i is the thermal conductivity of the i-th layer and $L_z = L_1 + L_2 + L_3$. It is also important to note that the origin of the depth scale is at the surface of the top layer and that all vertical distances are negative. For the steady-state situation, the power density may be written as

$$P(x, y) = U(x, y)P_0, \tag{26}$$

where the function $U(x, y)$ is the weighting function which describes the geometry and uniformity (or nonuniformity) of the heat-generating components, and P_0 is the steady-state power density per unit area. As discussed previously, the ambient temperature trivially satisfies the heat flow equation. Hence, it is convenient to consider all temperatures relative to ambient. Therefore, when the power density is set equal to zero, the temperature will be zero as it is relative to ambient. As the three layers are of the same lateral dimensions and there is no heat flow out of the lateral boundaries, the temperature in each of the three layers may be written in the form of eq (10). Further, the z-dependent Fourier expansion coefficients will be of the form of eq (17) and may be written as

$$\tau_1(n, m, z) = \alpha_1 \cosh(\gamma z) + \beta_1 \sinh(\gamma z), \tag{27}$$

$$\tau_2(n, m, z) = \alpha_2 \cosh(\gamma z) + \beta_2 \sinh(\gamma z), \tag{28}$$

$$\tau_3(n,m,z) = \alpha_3 \cosh(\gamma z) + \beta_3 \sinh(\gamma z). \tag{29}$$

There are six unknowns involved in these solutions and they are specifically the set of expansion coefficients $\alpha_i, \beta_i (i = 1,2,3)$. These coefficients may be explicitly evaluated by means of the boundary conditions on the temperature and its derivative evaluated at the interfaces. The Fourier coefficients may be used directly in the above boundary conditions as the temperature is a sum over the Fourier expansion coefficients. By substituting the above equations into the appropriate boundary condition equations, it is possible to obtain a system of six equations in six unknowns. For the present case, this system reduces to one with four equations in four unknowns. The standard method for solving this system is to use Cramer's rule [4] with the Laplace method [4] for the evaluation of the determinants involved. However, instead of using this method explicitly here, the Fourier coefficients for each of the layers will be presented and will be shown to satisfy the heat flow equation and the appropriate boundary conditions. In particular, the Fourier coefficients in the three layers are given by eqs (15-17) in Kokkas' paper. Specializing these to the steady-state situation, these are

$$\tau_1(n,m,z) = A\Big\{B\cosh\big(\gamma(L_1+z)\big) + C\sinh\big(\gamma(L_1+z)\big)\Big\}, \tag{30}$$

$$\tau_2(n,m,z) = A\Big\{D\cosh\big(\gamma(L_1+L_2+z)\big) + E\sinh\big(\gamma(L_1+L_2+z)\big)\Big\}, \tag{31}$$

$$\tau_3(n,m,z) = A\sinh\big(\gamma(L_z+z)\big), \tag{32}$$

where

$$A = \frac{U(n,m)P_0}{\kappa_1\gamma}\left\{\frac{1}{B\sinh(\gamma L_1) + C\cosh(\gamma L_1)}\right\}, \tag{33}$$

$$B = D\cosh(\gamma L_2) + E\sinh(\gamma L_2), \tag{34}$$

$$C = \frac{\kappa_2}{\kappa_1}\Big\{D\sinh(\gamma L_2) + E\cosh(\gamma L_2)\Big\}, \tag{35}$$

$$D = \sinh(\gamma L_3), \tag{36}$$

$$E = \frac{\kappa_3}{\kappa_2}\cosh(\gamma L_3), \tag{37}$$

and

$$U(n,m) = \int_0^{L_x}\int_0^{L_y} U(x,y)\cos(n\pi x/L_x)\cos(m\pi y/L_y)dxdy, \tag{38}$$

is the double Fourier cos transform of the power density uniformity function. Now, it will be shown that the above are the solutions of the equation (see eq (16))

$$\frac{\partial^2}{\partial z^2}\tau_i(n,m,z) - \gamma^2\tau_i(n,m,z) = 0, \tag{39}$$

where the subscript i takes on the values of 1,2,3. This may be easily shown to be the case as

$$\frac{\partial^2}{\partial z^2}\cosh\big(\gamma(L+z)\big) = \gamma^2\cosh\big(\gamma(L+z)\big), \tag{40}$$

and

$$\frac{\partial^2}{\partial z^2}\sinh\big(\gamma(L+z)\big) = \gamma^2\sinh\big(\gamma(L+z)\big), \tag{41}$$

where L is a constant and is equal to L_1, $L_1 + L_2$, or L_z in eqs (30-32). Hence, eqs (30-32) satisfy the z-dependent differential equation. Next, it will be shown that these Fourier coefficients satisfy the appropriate boundary conditions. The first of

these is that

$$\kappa_1 \frac{\partial \tau_1(n,m,z)}{\partial z}\Big|_{z=0} = P(n,m) = U(n,m)P_0. \tag{42}$$

Using the Fourier coefficient given by eq (30) for the top layer, this may be evaluated as

$$\kappa_1 \frac{\partial \tau_1(n,m,z)}{\partial z}\Big|_{z=0} = \kappa_1 \gamma A\Big\{B\sinh(\gamma L_1) + C\cosh(\gamma L_1)\Big\}$$

$$= \kappa_1 \gamma \frac{U(n,m)P_0}{\kappa_1 \gamma}\left\{\frac{1}{B\sinh(\gamma L_1) + C\cosh(\gamma L_1)}\right\}\Big\{B\sinh(\gamma L_1) + C\cosh(\gamma L_1)\Big\}$$

$$= U(n,m)P_0. \tag{43}$$

Hence, the top layer boundary condition is satisfied by the $\tau_1(n,m,z)$. Next, consider the bottom layer boundary condition, i.e.,

$$\tau_3(n,m,z)|_{z=-L_z} = 0. \tag{44}$$

Making use of eq (32), this may be readily evaluated as

$$\tau_3(n,m,z)|_{z=-L_z} = A\sinh\big(\gamma(L_z - L_z)\big) = 0. \tag{45}$$

Hence, the last boundary condition is satisfied. The final set of boundary conditions to be verified are the ones which pertain to the interface boundary conditions. These are

$$\tau_1(n,m,z)|_{z=-L_1} = \tau_2(n,m,z)|_{z=-L_1}, \tag{46}$$

$$\tau_2(n,m,z)|_{z=-(L_1+L_2)} = \tau_3(n,m,z)|_{z=-(L_1+L_2)}, \tag{47}$$

$$\kappa_1 \frac{\partial \tau_1(n,m,z)}{\partial z}\Big|_{z=-L_1} = \kappa_2 \frac{\partial \tau_2(n,m,z)}{\partial z}\Big|_{z=-L_1}, \tag{48}$$

$$\kappa_2 \frac{\partial \tau_2(n,m,z)}{\partial z}\Big|_{z=-(L_1+L_2)} = \kappa_3 \frac{\partial \tau_3(n,m,z)}{\partial z}\Big|_{z=-(L_1+L_2)}. \tag{49}$$

In order to verify these equations, it is simplest to calculate the right- and left-hand sides of the equations and then compare them directly. The left-hand side of the first temperature continuity equation may be evaluated as

$$\tau_1(n,m,z)|_{z=-L_1} = A\Big\{B\cosh\big(\gamma(L_1-L_1)\big) + C\sinh\big(\gamma(L_1-L_1)\big)\Big\}$$

$$= AB = A\Big\{D\cosh(\gamma L_2) + E\sinh(\gamma L_2)\Big\}. \tag{50}$$

The right-hand side of the equation may be evaluated as

$$\tau_2(n,m,z)|_{z=-L_1} = A\Big\{D\cosh\big(\gamma(L_1+L_2-L_1)\big) + E\sinh\big(\gamma(L_1+L_2-L_1)\big)\Big\}$$

$$= A\Big\{D\cosh(\gamma L_2) + E\sinh(\gamma L_2)\Big\}. \tag{51}$$

Hence, the first of the temperature continuity equations satisfies the boundary condition. Next, consider the second temperature continuity equation.

$$\tau_2(n,m,z)|_{z=-(L_1+L_2)} = \tau_3(n,m,z)|_{z=-(L_1+L_2)}, \tag{52}$$

The left-hand side of the equation may be evaluated as

$$\tau_2(n,m,z)|_{z=-(L_1+L_2)} = A\Big\{D\cosh\big(\gamma(L_1+L_2-L_1-L_2)\big)+E\sinh\big(\gamma(L_1+L_2-L_1-L_2)\big)\Big\}$$

$$= AD = A\sinh(\gamma L_3). \tag{53}$$

The right-hand side is

$$\tau_3(n,m,z)|_{z=-(L_1+L_2)} = A\sinh\big(\gamma(L_z - L_1 - L_2)\big) = A\sinh(\gamma L_3). \tag{54}$$

Hence, the second temperature continuity equation is satisifed. The next equation to be verified is the first heat flow continuity equation, i.e.,

$$\kappa_1 \frac{\partial \tau_1(n,m,z)}{\partial z}\Big|_{z=-L_1} = \kappa_2 \frac{\partial \tau_2(n,m,z)}{\partial z}\Big|_{z=-L_1}. \tag{55}$$

Evaluating the left-hand side leads to

$$\kappa_1 \frac{\partial \tau_1(n,m,z)}{\partial z}\Big|_{z=-L_1} = \kappa_1\gamma A\Big\{B\sinh\big(\gamma(L_1 - L_1)\big) + C\cosh\big(\gamma(L_1 - L_1)\big)\Big\}$$

$$= \kappa_1\gamma AC = \kappa_2\gamma A\Big\{D\sinh(\gamma L_2) + E\cosh(\gamma L_2)\Big\}. \tag{56}$$

The right-hand side may be evaluated as

$$\kappa_2 \frac{\partial \tau_2(n,m,z)}{\partial z}\Big|_{z=-L_1} = \kappa_2\gamma A\Big\{D\sinh(\gamma L_2) + E\cosh(\gamma L_2)\Big\}. \tag{57}$$

The final equation to be evaluated is that for the heat flow continuity between the second and third layers, i.e.,

$$\kappa_2 \frac{\partial \tau_2(n,m,z)}{\partial z}\Big|_{z=-(L_1+L_2)} = \kappa_3 \frac{\partial \tau_3(n,m,z)}{\partial z}\Big|_{z=-(L_1+L_2)}. \tag{58}$$

The left-hand side of this equation may be evaluated as

$$\kappa_2 \frac{\partial \tau_2(n,m,z)}{\partial z}\Big|_{z=-(L_1+L_2)}$$

$$= \kappa_2\gamma A\Big\{D\sinh\big(\gamma(L_1 + L_2 - L_1 - L_2)\big) + E\cosh\big(\gamma(L_1 + L_2 - L_1 - L_2)\big)\Big\}$$

$$= \kappa_2 \gamma AE = \kappa_3 \gamma A \cosh(\gamma L_3). \tag{59}$$

The right-hand side is

$$\kappa_3 \frac{\partial \tau_3(n,m,z)}{\partial z}\Big|_{z=-(L_1+L_2)} = \kappa_3 \gamma A \cosh\big(\gamma(L_z - L_1 - L_2) = \kappa_3 \gamma A \cosh(\gamma L_3). \tag{60}$$

Now that it has been shown that the Fourier coefficients satisfy the steady-state heat flow problem and the appropriate boundary conditions, there are several points to be considered before getting into the body of the program. These include: (1) evaluation of the function $U(n,m)$ for a uniform power source of given size and (2) simplification of the Fourier coefficients for subsequent numerical analysis. The latter point is necessary as the limits of γ very small and γ very large may give rise to overflow or underflow problems when the program is constructed.

FORM OF THE FUNCTION U(n,m)

The first thing to be considered is the form of the function $U(n,m)$ for an arbitrary number of heat sources. This function is defined as

$$U(n,m) = \int_0^{L_x} \int_0^{L_y} U(x,y) \cos(n\pi x/L_x) \cos(m\pi y/L_y) dx dy. \tag{61}$$

The analysis can most easily be accomplished in terms of a single uniform heat source. The case of an arbitrary number of heat sources follows by summing the results of each heat source. Further, if any of the heat sources are nonuniform, their effects can be constructed by suitably overlapping a number of uniform heat sources. In the coordinate system being used, consider a single heat source denoted by the index i with a corner at the location (x_i, y_i) and lengths along the x- and y-directions given by (lx_i, ly_i). Over the area of the heat source, $U(x,y)$ is assumed to be uniform and equal to unity. Away from the area of the heat source, $U(x,y)$ is assumed to be equal to zero. Then, $U(x,y)$ may be viewed as being a unit step function over the surface of the power source. Consequently, the contribution from this single heat source may be written as

$$\int_0^{L_x} \int_0^{L_y} U(x,y) \cos(n\pi x/L_x) \cos(m\pi y/L_y) dx dy =$$

$$\int_{x_i}^{x_i+lx_i} \int_{y_i}^{y_i+ly_i} \cos(n\pi x/L_x) \cos(m\pi y/L_y) dx dy. \tag{62}$$

The integrals can be simply evaluated to give the result that

$$U_i(n,m) = \frac{L_x L_y}{(n\pi)(m\pi)}\left\{\sin\left(\frac{n\pi(x_i + lx_i)}{L_x}\right) - \sin\left(\frac{n\pi x_i}{L_x}\right)\right\}$$

$$\times\left\{\sin\left(\frac{m\pi(y_i + ly_i)}{L_y}\right) - \sin\left(\frac{m\pi y_i}{L_y}\right)\right\}. \tag{63}$$

Similar expressions may be written for each of the heat sources and then summed to give the cumulative heat source effect. In addition, as indicated previously, if there are any nonuniform heat sources, their effect can be constructed by means of overlapping uniform heat sources. Before turning to the small γ and large γ behavior of the Fourier coefficients, it is important to consider the behavior of the function $U(n,m)$ for either $n = 0$, $m = 0$, or both. This is important in the numerical implementation of the solutions as the program will have to calculate the double Fourier cos transform over the range of n, m required by the sum in eq (10). Once the value of n or m is zero, there will be problems with most machines as far as evaluating the seeming divergence. This can be circumvented by investigating the behavior of the function for $n = 0$. This may be readily carried out by first noting that the function $U(n,m)$ (for the particular form of $U(x,y)$) is a product of two terms. This may be simply written as $U(n,m) = U(n)U(m)$. Then, consider the n (from the x integration) contribution to the function which is given by

$$U(n) = \frac{L_x}{n\pi}\left\{\sin\left(\frac{n\pi(x_i + lx_i)}{L_x}\right) - \sin\left(\frac{n\pi x_i}{L_x}\right)\right\}. \tag{64}$$

There is an apparent divergence or infinity if n is simply set equal to zero. This is the way in which a computer would look at the expression. However, this infinity is not real as can be seen by using the expansion of the sin function for small values

of the argument. In particular,

$$\sin(x) = x - \frac{x^3}{3!} + \cdots. \tag{65}$$

Making use of this expression for the sin function, it is straightforward to show that

$$\lim_{n\to 0} U(n) = \lim_{n\to 0} \frac{L_x}{n\pi}\left\{\sin\left(\frac{n\pi(x_i + lx_i)}{L_x}\right) - \sin\left(\frac{n\pi x_i}{L_x}\right)\right\} =$$

$$\lim_{n\to 0} \frac{L_x}{n\pi}\left\{\left(\frac{n\pi(x_i + lx_i)}{L_x}\right) - \left(\frac{n\pi x_i}{L_x}\right)\right\} = lx_i. \tag{66}$$

The same conclusion holds for the y-dependent portion, i.e., $U(m)$. This must be specially coded to bypass any overflow problem. The specific coding of the heat source may be found in the program listing in the function UZERO.

In general, the function $U(n, m)$ is oscillatory and does not approach zero sufficiently fast for large values of n or m. In particular, eq (63) shows that $U_i(n, m) \to 0$ like $1/nm$ as $m, n \to \infty$. In addition, the cos terms in eq (10), i.e., $\cos(n\pi x/L_x)$ $\cos(m\pi y/L_y)$, do not approach a definite limit for large values of the argument. Consequently, the product $U(n, m)\cos(n\pi x/L_x)\cos(m\pi y/L_y)$ tends to zero slowly. An example of the behavior of the x-dependent portion of this product function, i.e., $U(n)\cos(n\pi x/L_x)$ is presented in figure 2. The heat source used for this figure is a 1 mil by 1 mil heat source centered on the surface of a 200 mil by 200 mil structure. The product function is evaluated at the midpoint of the heat source. Because of the symmetry used, the function shows damped oscillatory behavior. It is clear from the figure that the product function does not fall off to zero sufficiently fast and hence does not provide for rapid convergence of the calculated temperature using the Fourier representation.

BEHAVIOR OF FOURIER COEFFICIENTS:

SMALL VALUES OF THE ARGUMENT

As has been seen in the treatment of the function $U(n,m)$, care must be taken for the case where both n and m are equal to zero (or $\gamma = 0$). The same considerations must be carried out for the Fourier coefficients in the three layers. In the solutions in the three layers, the summation indices n, m appear. The summation over these variables is of influence in the variable γ according to eq (15). Also, the Fourier coefficients contain the hyperbolic functions which depend upon γ. For large values of γ, the sinh and cosh functions grow exponentially. This can present special numerical problems when the summation variables approach the upper limits which may be required for the case of very small heat sources. Hence, special care must be taken to study the behavior of the Fourier coefficients for small γ and large γ so as to remove any potential numerical overflow problems. Once this is properly taken care of, the Fourier coefficients and the solutions will be numerically well behaved.

First, consider the small γ behavior of the Fourier coefficients. This is done by considering the small γ behavior of the eqs (30-37) and the small argument behavior of the hyperbolic functions. In the following discussion as well as the discussion of the large γ behavior, the term $U(n,m)P_0/\kappa_1$ will be removed for convenience. This term will henceforth be included explicitly in the sum in eq (10) as the Fourier coefficients for all three layers contain this as a common factor through A (see eqs (30-33)). Then, for small γ,

$$E \approx \frac{\kappa_3}{\kappa_2}, \tag{67}$$

$$D \approx \gamma L_3, \tag{68}$$

$$C \approx \frac{\kappa_2}{\kappa_1}\left\{\gamma^2 L_2 L_3 + \frac{\kappa_3}{\kappa_2}\right\}, \tag{69}$$

$$B \approx \gamma L_3 + \frac{\kappa_3}{\kappa_2}\gamma L_2, \tag{70}$$

and, remembering that the factor $U(n,m)P_0/\kappa_1$ has been included explicitly in eq (10),

$$A \approx \frac{1}{\gamma}\frac{\kappa_1}{\kappa_3}. \tag{71}$$

Making use of these expressions and the small argument behavior of the hyperbolic functions, it is straightforward to investigate the small γ behavior of the solutions. In particular,

$$\tau_1(n,m,z) \approx \frac{1}{\gamma}\frac{\kappa_1}{\kappa_3}\left\{\gamma L_3 + \frac{\kappa_3}{\kappa_2}\gamma L_2 + \frac{\kappa_2}{\kappa_1}\left\{\gamma^2 L_2 L_3 + \frac{\kappa_3}{\kappa_2}\right\}\gamma(L_1 + z)\right\}$$

$$\approx \frac{\kappa_1}{\kappa_3}\left\{L_3 + \frac{\kappa_3}{\kappa_2}L_2 + \frac{\kappa_3}{\kappa_1}(L_1 + z)\right\}.$$

Then,

$$\lim_{\gamma\to 0} \tau_1(n,m,z) = (L_1 + z) + \frac{\kappa_1}{\kappa_2}L_2 + \frac{\kappa_1}{\kappa_3}L_3. \tag{72}$$

Next,

$$\tau_2(n,m,z) \approx \frac{1}{\gamma}\frac{\kappa_1}{\kappa_3}\left\{\gamma L_3 + \frac{\kappa_3}{\kappa_2}\gamma(L_1 + L_2 + z)\right\},$$

or

$$\lim_{\gamma\to 0} \tau_2(n,m,z) = \frac{\kappa_1}{\kappa_3}L_3 + \frac{\kappa_1}{\kappa_2}(L_1 + L_2 + z). \tag{73}$$

And finally,

$$\tau_3(n,m,z) \approx \frac{1}{\gamma}\frac{\kappa_1}{\kappa_3}\Big\{\gamma(L_1+L_2+L_3+z)\Big\},$$

or

$$\lim_{\gamma\to 0}\tau_3(n,m,z) = \frac{\kappa_1}{\kappa_3}(L_1+L_2+L_3+z). \tag{74}$$

These special forms of the Fourier coefficients (in the limit as $\gamma \to 0$) are necessary in the code to bypass overflow problems for small values of the argument.

BEHAVIOR OF FOURIER COEFFICIENTS:

LARGE VALUES OF THE ARGUMENT

As the Fourier coefficients have been investigated for small γ and have been shown to be well behaved when properly written, what remains is to write these coefficients in a form which is amenable for investigating their large γ behavior. As noted before, the hyperbolic functions, sinh and cosh, grow exponentially for large values of the argument. On the other hand, the hyperbolic tanh approaches unity for large values of the argument. With this in mind, let us investigate the form of the Fourier coefficients, written as much as possible in terms of the tanh, which takes care of this potential numerical difficulty. To this end, it is convenient to introduce the shorthand notation for the hyperbolic functions, $c(x) = \cosh(x)$, $s(x) = \sinh(x)$, and $t(x) = \tanh(x)$. Making use of this shorthand notation, the Fourier coefficients may be written as

$$\tau_1(n,m,z) = A\Big\{Bc\big(\gamma(L_1+z)\big) + Cs\big(\gamma(L_1+z)\big)\Big\}, \tag{75}$$

$$\tau_2(n,m,z) = A\Big\{Dc\big(\gamma(L_1+L_2+z)\big) + Es\big(\gamma(L_1+L_2+z)\big)\Big\} \tag{76}$$

$$\tau_3(n,m,z) = As(\gamma(L_z+z). \tag{77}$$

where,

$$A = \frac{1}{\gamma}\left\{\frac{1}{Bs(\gamma L_1) + Cc(\gamma L_1)}\right\}, \tag{78}$$

$$B = Dc(\gamma L_2) + Es(\gamma L_2), \tag{79}$$

$$C = \frac{\kappa_2}{\kappa_1} \; D s(\gamma L_2) + E c(\gamma L_2) \;\; , \tag{80}$$

$$D = s(\gamma L_3), \tag{81}$$

$$E = \frac{\kappa_3}{\kappa_2} c(\gamma L_3). \tag{82}$$

As in the investigation of the small γ behavior of the Fourier coefficients, the factor $U(n,m)P_0/\kappa_1$ has been deleted from eq (78) for convenience. This factor may simply be included in the Fourier representation of the temperature, eq (10), as it is common to all three layers. Now, the above equations (eqs (75-82)) will be rewritten by making use of the definition of the hyperbolic tanh, i.e., $t(x) = s(x)/c(x)$. First, consider the coefficient C.

$$C = \frac{\kappa_2}{\kappa_1} \Big\{ D s(\gamma L_2) + E c(\gamma L_2) \Big\}$$

$$C = \frac{\kappa_2}{\kappa_1} \Big\{ s(\gamma L_3) s(\gamma L_2) + \frac{\kappa_3}{\kappa_2} c(\gamma L_3) c(\gamma L_2) \Big\}$$

$$C = \frac{\kappa_2}{\kappa_1} c(\gamma L_3) c(\gamma L_2) \Big\{ t(\gamma L_3) t(\gamma L_2) + \frac{\kappa_3}{\kappa_2} \Big\}. \tag{83}$$

Next, the coefficient B may be written as

$$B = D c(\gamma L_2) + E s(\gamma L_2)$$

$$B = s(\gamma L_3) c(\gamma L_2) + \frac{\kappa_3}{\kappa_2} c(\gamma L_3) s(\gamma L_2)$$

$$B = c(\gamma L_3) c(\gamma L_2) \Big\{ t(\gamma L_3) + \frac{\kappa_3}{\kappa_2} t(\gamma L_2) \Big\}. \tag{84}$$

Then,

$$Bs(\gamma L_1) + Cc(\gamma L_1) =$$

$$c(\gamma L_3)c(\gamma L_2)s(\gamma L_1)\left\{t(\gamma L_3)+\frac{\kappa_3}{\kappa_2}t(\gamma L_2)\right\}+c(\gamma L_3)c(\gamma L_2)c(\gamma L_1)\frac{\kappa_2}{\kappa_1}\left\{t(\gamma L_3)t(\gamma L_2)+\frac{\kappa_3}{\kappa_2}\right\}$$

$$= c(\gamma L_3)c(\gamma L_2)c(\gamma L_1)\left\{t(\gamma L_3)t(\gamma L_1)+\frac{\kappa_3}{\kappa_2}t(\gamma L_1)t(\gamma L_2)+\frac{\kappa_2}{\kappa_1}t(\gamma L_3)t(\gamma L_2)+\frac{\kappa_3}{\kappa_1}\right\}. \quad (85)$$

Then, the coefficient A may be written as

$$A =$$

$$\frac{1}{\gamma c(\gamma L_3)c(\gamma L_2)c(\gamma L_1)}\frac{1}{\left\{t(\gamma L_3)t(\gamma L_1)+\frac{\kappa_3}{\kappa_2}t(\gamma L_1)t(\gamma L_2)+\frac{\kappa_2}{\kappa_1}t(\gamma L_3)t(\gamma L_2)+\frac{\kappa_3}{\kappa_1}\right\}}. \quad (86)$$

It is convenient to define the function $\Omega(\gamma)$ as

$$\Omega(\gamma) = \frac{1}{\left\{t(\gamma L_3)t(\gamma L_1)+\frac{\kappa_3}{\kappa_2}t(\gamma L_1)t(\gamma L_2)+\frac{\kappa_2}{\kappa_1}t(\gamma L_3)t(\gamma L_2)+\frac{\kappa_3}{\kappa_1}\right\}}, \quad (87)$$

which is well behaved for all values of γ. Then, the coefficient A may be written as

$$A = \frac{\Omega(\gamma)}{\gamma c(\gamma L_3)c(\gamma L_2)c(\gamma L_1)}. \quad (88)$$

By making use of the above procedure, it is relatively straightforward to show that the Fourier coefficients as given by eqs (75-77) may be written as

$$\tau_1(n,m,z) =$$

$$\frac{\Omega(\gamma)c\big(\gamma(L_1+z)\big)}{\gamma c(\gamma L_1)}\left\{t(\gamma L_3)+\frac{\kappa_3}{\kappa_2}t(\gamma L_2)+t\big(\gamma(L_1+z)\big)\frac{\kappa_2}{\kappa_1}\left(t(\gamma L_3)t(\gamma L_2)+\frac{\kappa_3}{\kappa_2}\right)\right\}, \quad (89)$$

$$\tau_2(n,m,z) = \frac{\Omega(\gamma)c\big(\gamma(L_1+L_2+z)\big)}{\gamma c(\gamma L_1)c(\gamma L_2)}\left\{t(\gamma L_3)+\frac{\kappa_3}{\kappa_2}t\big(\gamma(L_1+L_2+z)\big)\right\}, \quad (90)$$

and

$$\tau_3(n,m,z) = \frac{\Omega(\gamma)s\big(\gamma(L_1+L_2+L_3+z)\big)}{\gamma c(\gamma L_1)c(\gamma L_2)c(\gamma L_3)}. \quad (91)$$

In eqs (89-91), the function $\Omega(\gamma)$ and the terms inside the curly brackets are well behaved for all values of the variable, γ. The sinh and cosh terms which remain may still give rise to numerical overflow problems for large values of the argument. However, as both of these functions grow exponentially for large values of the argument and they appear in both the numerator and the denominator of the Fourier coefficients, there will be cancellation. This cancellation for large values of the argument will not be worked out in detail here but is contained in the FORTRAN listing of the program in the function FUNZ. These Fourier coefficients are used in the equation

$$T_i(x,y,z) = P_0\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{4U(n,m)\tau_i(n,m,z)\cos(n\pi x/L_x)\cos(m\pi y/L_y)}{(\delta_{n0}+1)(\delta_{m0}+1)L_xL_y\kappa_1} \quad (92)$$

for $i = 1,2,3$ to obtain the solutions in each of the three layers. In the above, the term $P_0U(n,m)/\kappa_1$ has been written out explicitly and is no longer contained in

the Fourier coefficients. It is important in obtaining the solution in x, y, z to use the appropriate layer equation. This is automatically taken care of in the program as the depth z is compared with the various thicknesses and the corresponding layer Fourier coefficient is used. The user does not have to specify which layer is to be used.

An interesting exercise left to the reader is to show when the three thermal conductivities are equal that the one-layer solution is obtained. Also, another exercise is to show that the one-layer solution is obtained when the thicknesses of the second and third layers are set equal to zero.

In the subsequent discussion, it is important to keep in mind that the Fourier coefficients are functions of the variables n and m. As discussed in the previous sections, the power density function $U(n, m)$ tends to zero very slowly for large values of the argument. Also, the cos terms in eq (92) do not tend to any limit as the arguments approach infinity. It is the Fourier coefficients which are responsible for the convergence of the sum. Consequently, some discussion of the n and m behavior of the Fourier coefficients is warranted. To show how the Fourier coefficients behave as a function of n and m (or γ), these functions were studied in some detail. In figure 3, a three-dimensional plot of the n and m dependence of the top layer Fourier coefficient is shown at $z = 0$ for a three-layer structure. In the figure, it can be seen that the Fourier coefficients are peaked at the origin of the n, m plane and approach zero as n and m become large. In addition, figure 4 contains the same type of plot for the situation where all three layers have the same thermal conductivity, i.e., for a thick one-layer structure.

The dependence on n of the three Fourier coefficients at the top of each layer is contained in figure 5 for the structure used to generate figure 3. The curve marked by A shows the behavior on the top surface of the structure, i. e., at $z = 0$. The curves denoted by B and C present the results for the top surfaces of the second and third layers, respectively. Figure 6 is a log-log plot form of figure 5a and is especially important as it shows the slow convergence of the top surface Fourier coefficient which will be discussed in detail in the section on the effects of the upper limit on the calculated temperature.

SPECIAL CASE OF POWER SOURCE COVERING TOP SURFACE

As a special case of eq (92), consider the situation of a single power source completely covering the top surface; i.e., there is a single heat source with lateral dimensions equal to that of the three-layer structure. In this particular case, it will be shown that the above equation reduces to the familiar thermal resistance equation. The easiest way to proceed with the analysis is to consider the specific form of the function $U(n,m)$. From eq (63),

$$U_1(n,m) =$$

$$\frac{L_x L_y}{(n\pi)(m\pi)}\left\{\sin\left(\frac{n\pi(x_1+lx_1)}{L_x}\right)-\sin\left(\frac{n\pi x_1}{L_x}\right)\right\}\left\{\sin\left(\frac{m\pi(y_1+ly_1)}{L_y}\right)-\sin\left(\frac{m\pi y_1}{L_y}\right)\right\}. \tag{93}$$

For the particular case of uniform surface coverage, $x_1 = y_1 = 0$, $lx_1 = L_x$ and $ly_1 = L_y$. Upon substituting these values into the equation, the function reduces to

$$U_1(n,m) = \frac{L_x L_y}{(n\pi)(m\pi)}\Big\{\sin(n\pi)\sin(m\pi)\Big\}. \tag{94}$$

This is zero when the indices are nonzero. For the case where both of the indices are zero, the use of the expansion of the sin function gives rise to the result that

$$U_1(n,m) = L_x L_y \delta_{n0}\delta_{m0}. \tag{95}$$

If this form of the $U(n,m)$ function is substituted into the equation for the temperature in each of the three layers (eq (92)) and the form of the Fourier expansion coefficients as $\gamma \to 0$ (eqs (72-74)) is used, it is readily shown that the temperatures

in the three layers may be written as

$$T_1(x, y, z) = P_0\left\{\frac{L_1 + z}{\kappa_1} + \frac{L_2}{\kappa_2} + \frac{L_3}{\kappa_3}\right\} \tag{95}$$

$$T_2(x, y, z) = P_0\left\{\frac{L_1 + L_2 + z}{\kappa_2} + \frac{L_3}{\kappa_3}\right\} \tag{96}$$

$$T_3(x, y, z) = P_0\left\{\frac{L_1 + L_2 + L_3 + z}{\kappa_3}\right\}. \tag{97}$$

As P_0 is the power density per unit area, these equations give rise to the usual results of the one-dimensional calculations of the thermal resistance.

EFFECT OF UPPER SUMMATION LIMITS ON TEMPERATURE

Previously, the dependence of the Fourier coefficients on n and m has been discussed. Figures 3 to 6 contain typical results of this behavior. The purpose of the present section is to show how this behavior is mirrored in the calculated temperature. In particular, eqs (89-91) will be used in eq (92) with a variable upper summation limit in the latter equation. To simplify the analysis, a single heat source with $y_i = 0$ and $ly_i = Ly$ will be used. The width of this stripe heat source will be $lx_i = 1$ mil. Finally, the lateral dimensions of the three-layer structure will be taken as $L_x = L_y = 200$ mil. This particular choice will require only the $m = 0$ term in the sum to be retained while needing a large value of n terms in order to obtain convergence of the sum. As there is complete coverage along the y-direction, the m-dependent portion of eq (95) may be used in eq (92) to obtain

$$T_i^N(x, y, z) = P_0 \sum_{n=0}^{N} \frac{2U(n)\tau_i(n, 0, z)\cos(n\pi x/L_x)}{(\delta_{n0} + 1)L_x\kappa_1}, \tag{98}$$

where the dependence of $T_i^N(x, y, z)$ on N, the upper limit of summation, is considered in this section. Curves A, B, and C in figure 7 show the temperature calculated at the center of the heat source as a function of the number of terms in the sum, N, for the top of the first, second, and third layers, respectively. It is clear from the curve in figure 7a that the calculation of the surface temperature (at $z = 0$) may require up to at least 350 terms in the sum while, from figures 7b and 7c, the temperature below the surface may need only 20 terms to be retained in the sum. From the curve depicted in figure 7a, it may be argued that the number of terms needed to adequately represent a feature size of Δx (typical of the lateral size of a heat source) should be on the order of $L_x/\Delta x$. For the present case, this ratio

is about 200. Deeper into the structure, the heat flow has caused the approximate size of the heat current to spread out and hence fewer terms are necessary. This is borne out by the behavior of the temperature in the figure.

GENERAL DISCUSSION OF THE TXYZ PROGRAM

The annotated listing of the program is contained in the appendix of this report. In the present section, several aspects of the program will be discussed. The purpose of this discussion is to present the user with information concerning the implementation and/or modification of the program.

The first item is the format in which input data are to be entered. As listed in the program, data are to be entered in a fixed-field format. However, many FORTRAN compilers support free-field or list-directed read statements. Free-field read statements are convenient as they allow the user to change input data without having to be concerned with the tedious process of lining up of the data required by fixed-field read statements. Hence, if the user's FORTRAN compiler allows for free-field read statements, it is strongly suggested that the program be edited such that all read statements are of this form.

The next item is concerned with the situation of the hyperbolic functions being or not being built-in functions. If these functions are contained in the computer's relocatable (or object code) mathematics library, then there are no changes required in the code. If, however, these are not, then it is necessary to write separate function subroutines for the hyperbolic functions. As the hyperbolic functions are easily generated from the exponential function, this is a straightforward process.

Next, it is important to discuss the number of heat sources and the dimension statements used in the program. In its present form, TXYZ will allow up to twenty heat sources. This was deemed sufficient for most problems of interest. However, should the user want to use more than this number of heat sources (especially

for the case of a number of nonuniform heat sources), it is necessary to change the corresponding dimension statements. This is commented upon in the program and the change is relatively straightforward. The only other comment concerning dimension statements has to do with the number of points at which the temperature is to be calculated and the maximum number of terms to be used in the calculation of the temperature using eq (92). The program allows up to 501 as the maximum number for both of these quantities. This was considered to be adequate for the resolution of the spatial variation of the temperature and for the calculation of the temperature (even for the smallest physically realizable heat source on a structure of typical lateral dimensions). Hence, it is felt that 501 is a good maximum value which gives sufficient detail without excessive use of computer memory space. Should memory allocation be a problem, it may become necessary to reduce these numbers in the dimension statements. It is important to carefully evaluate any reduction in the maximum number of terms in the Fourier representation of the temperature in light of the slow convergence of the surface temperature for small heat sources. Finally, in regard to changing the dimension statements, it is important to change the dimension statements in not only the main program but also in the function subroutines as the dimensioned variables are declared in common and hence must have the same dimensions everywhere (in the main program and the function subroutines) to avoid any register difficulties.

A copy of this FORTRAN program may be obtained from the author by sending a letter of request and a computer tape.

SELECTED PROGRAM EXAMPLES

In order to assist the user in the implementation of the TXYZ program, several examples are presented in this section. In particular, the input data files are listed and plots of the temperature profiles are presented in the accompanying figures. The input data files have been annotated in order to facilitate familiarity with the reading of these files. This is especially important because of the number of input variables which set various "switches" inside the program. The corresponding data files will be sent along with the FORTRAN listing of the TXYZ program so as to simplify the implementation of the program and its subsequent use.

Two examples will be discussed in this section. The first is for the situation of a single small heat source on the surface of the top layer. The second example is that of a number of heat sources on the top layer. In particular, one of the heat sources is nonuniform and its construction from uniform heat sources is illustrated in the ccorresponding input data file.

The first example is that of a 1 mil by 1 mil uniform heat source on the surface of a 150 mil by 200 mil three-layer structure. The specific structure is that of 15 mils of silicon on 2 mils of die attach material on 30 mils of a substrate. This three-layer structure will be used in both this example and the one to follow. The input data file for this first example is listed, with annotation, in Table 1. The calculated temperature is presented in figure 8.

The second example is for the same three-layer structure as used in the first example. However, in this case, there are several heat sources, with one of them being nonuniform and built up from uniform heat sources. In particular, there are three

square uniform heat sources on the right portion of the surface and one uniform heat source in the middle of the surface. The nonuniform heat source is a single rectangular area. The surface temperature calculated along the line $0 \leq x \leq 200$, $y = 101$ is presented in figure 9. The input data file is presented in Table 2.

In addition to these examples, the reader's attention is drawn to the recent review [5] where this program is used in the thermal evaluation of VLSI packages using test chips.

ACKNOWLEDGMENTS

The author would like to thank Frank F. Oettinger for his continued interest during the course of this work. His relentless use of the program brought many of the early bugs to the surface.

The author would also like to thank Stephen E. Ross who changed the code into a portable FORTRAN form. His assistance in getting the three-dimensional plots is also appreciated.

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1. Kokkas, A. G., Thermal Analysis of Multiple-Layer Structures, *IEEE Trans. Electron Devices* **ED-21**, 674-681 (1974).

2. Carslaw, H. S., and Jaeger, J. C., *Conduction of Heat in Solids*, Second Edition (Oxford University Press, 1959).

3. Churchill, R. V., *Fourier Series and Boundary Value Problems*, Second Edition (McGraw-Hill, 1963).

4. Ayres, F., Jr., *Matrices* (Schaum Publishing Co., 1950).

5. Oettinger, F. F., Thermal Evaluation of VLSI Packages Using Test Chips - A Critical Review, *Solid State Technology* **27 (2)**, 169-179 (1984).

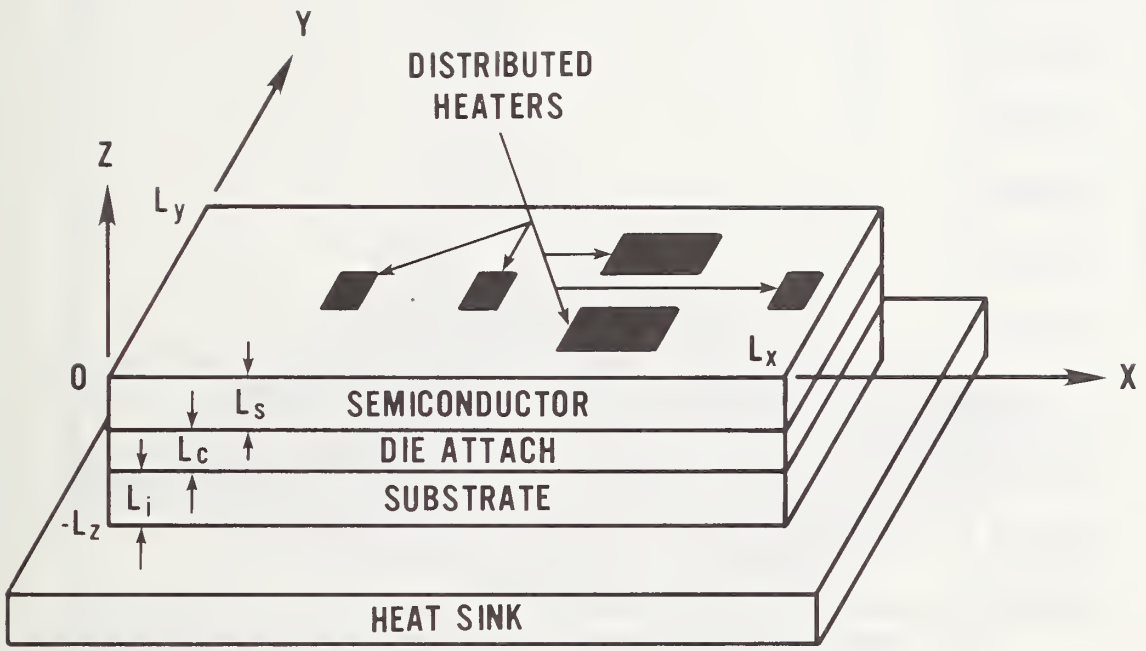

Figure 1. This figure presents the geometry of the three-layer structure in which the steady-state temperature is calculated.

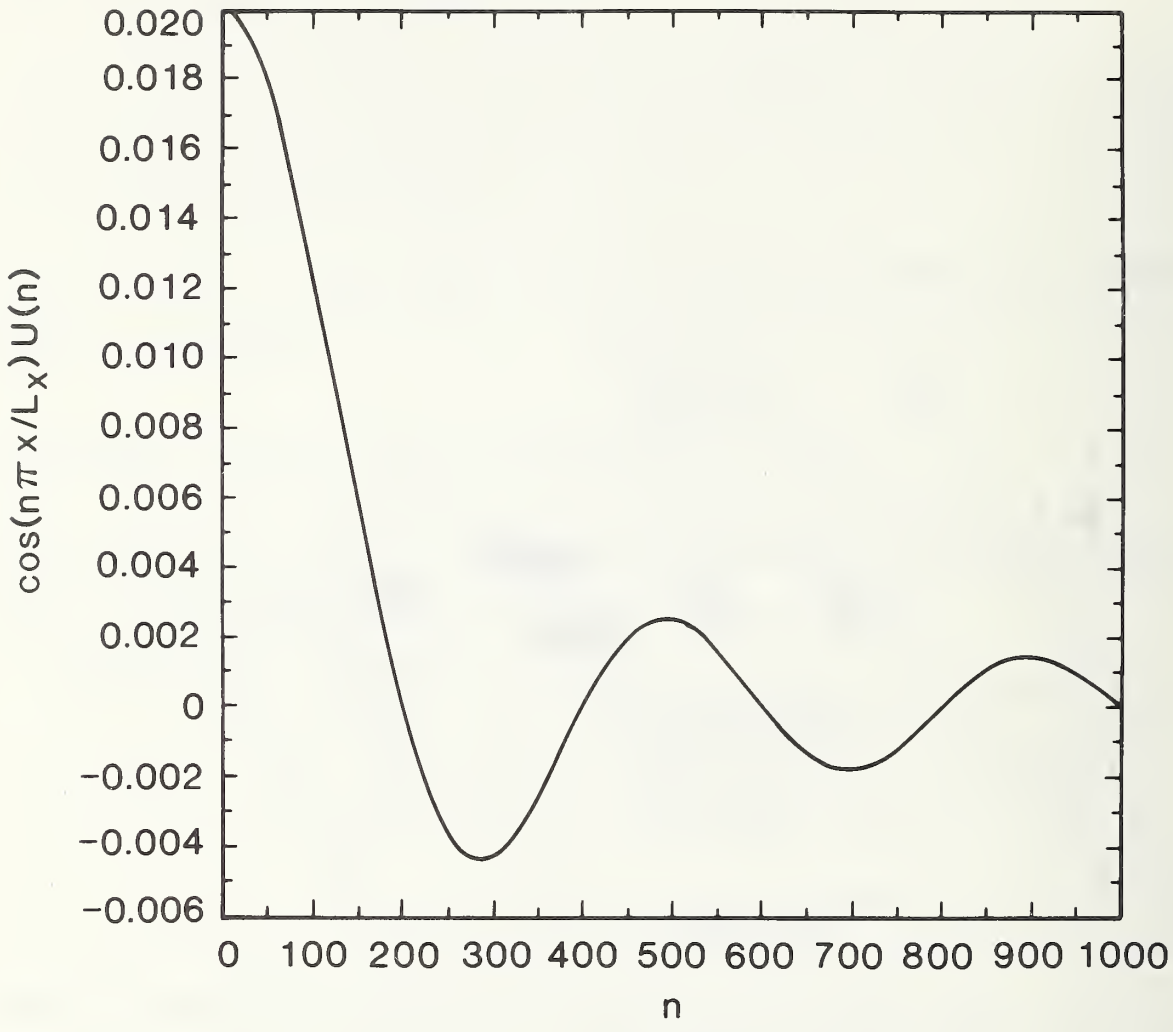

Figure 2. This figure shows the n-dependence of the product of the power density function and the cos terms in eq (92), i.e., $U(n)\cos(n\pi x/L_x)$, evaluated at the center of a single small heat source which is in the form of a thin stripe along the y-direction. This specific form of the product is used as there is complete coverage along the y-direction which means that only the $m = 0$ term contributes. The general feature seen in this figure is that this function does not asymptote to zero for n on the order of 500. This general kind of behavior has been found to be the case for all situations considered.

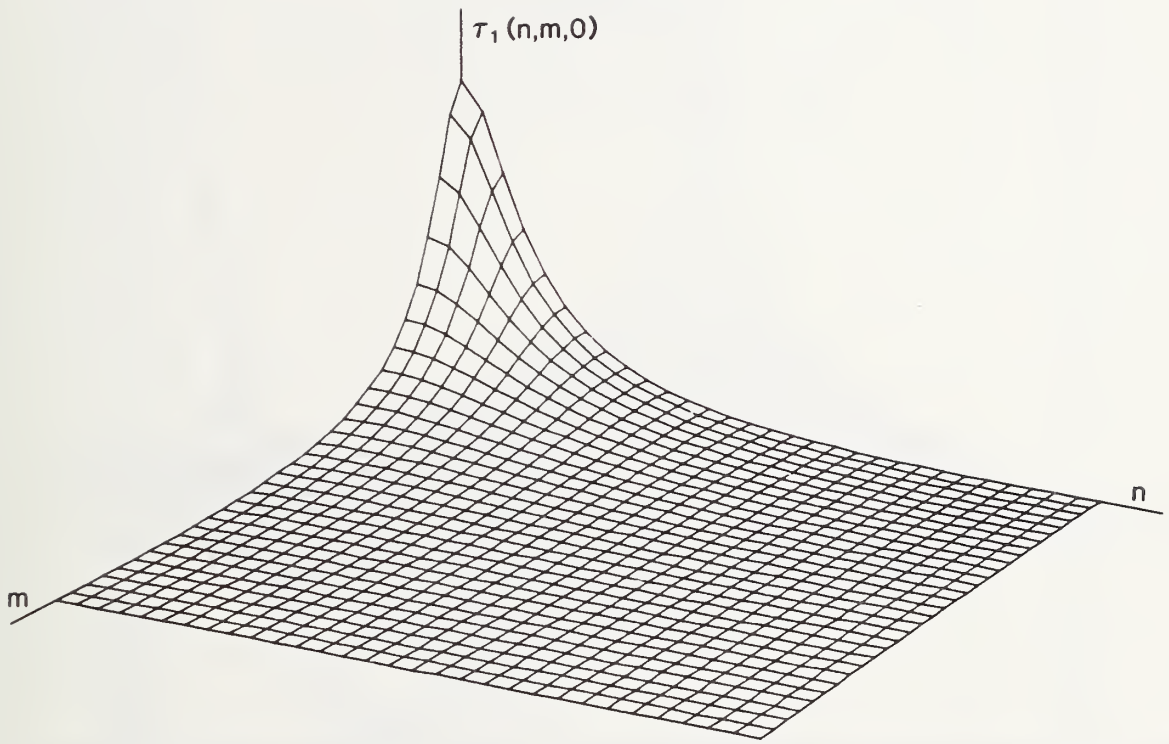

Figure 3. The top-layer Fourier coefficient, $\tau_1(n, m, 0)$, is shown as a function of n and m and at $z = 0$ for a three-layer structure where all three layers have different thermal conductivities. The specific structure is for 15 mils of silicon ($\kappa_1 = 0.00267$) over 2 mils of die attach ($\kappa_2 = 0.00064$) over 30 mils of substrate material ($\kappa_3 = 0.00999$). The peak value of τ_1 is at the origin of the n, m plane.

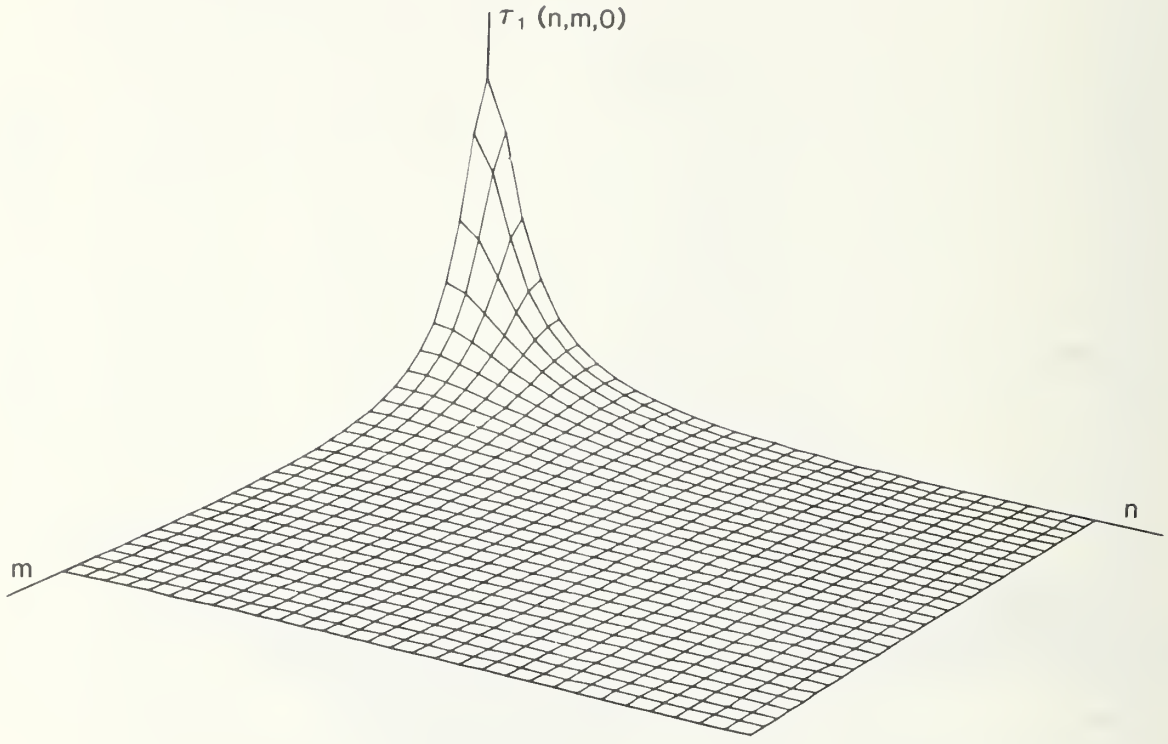

Figure 4. The top-layer Fourier coefficient, $\tau_1(n, m, 0)$, is shown as a function of n and m and at $z = 0$ for a three-layer structure where all three layers have the same thermal conductivity, i.e., a thick one-layer structure. The specific example is for 47 mils of silicon ($\kappa = 0.00267$). As with the previous figure, τ_1 is peaked at the origin of the n, m plane. Both this and the previous figure are meant to convey qualitative features of the Fourier coefficient. More detailed information will be presented in the next two figures where specific attention will be focused upon just the n axis (with $m = 0$) behavior.

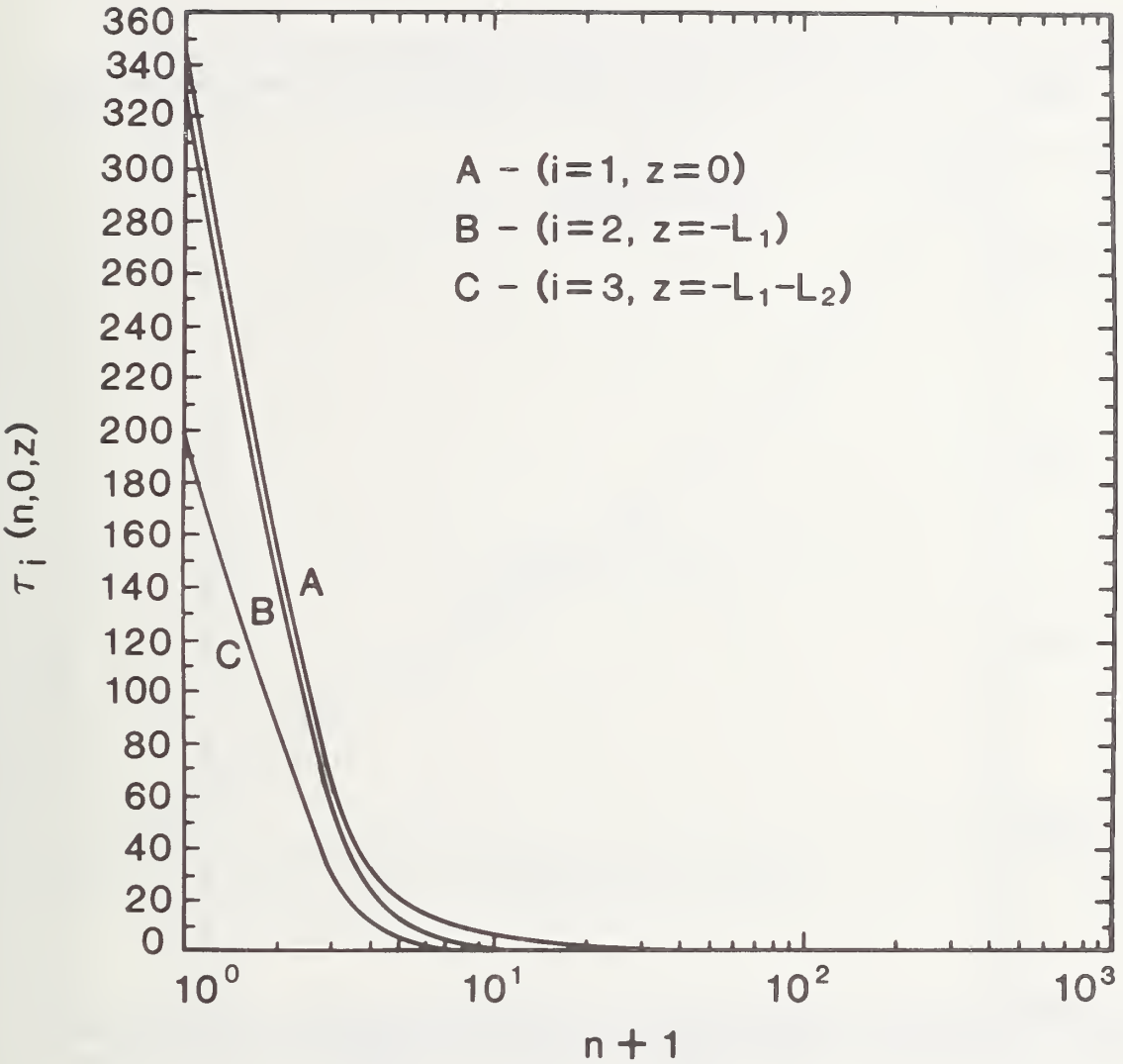

Figure 5. This figure shows the dependence of the three Fourier coefficients on n at the top of each of the layers. The curve denoted by A is $\tau_1(n, m = 0, z = 0)$ (for the top of the first layer). B shows the Fourier coefficient, $\tau_2(n, m = 0, z = -L_1)$ (for the top of the second layer). Finally, the curve denoted by C represents the Fourier coefficient, $\tau_3(n, m = 0, z = -L_1 - L_2)$, i.e., at the top of the third layer. This figure and the next represent a cross section along the n axis of figures of the type presented in figures 3 and 4.

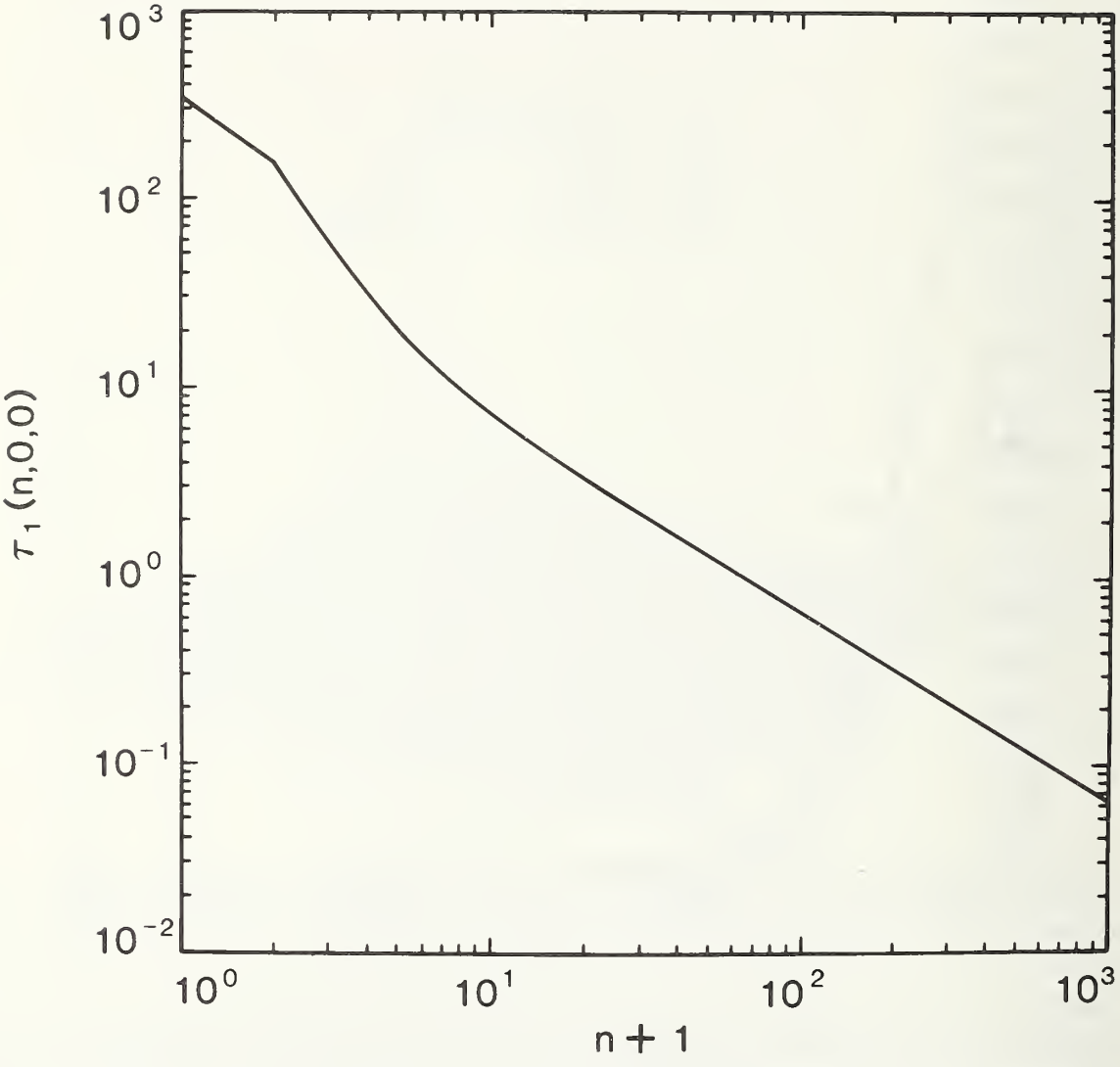

Figure 6. The behavior of the top layer Fourier coefficient, $\tau_1(n, m = 0, z = 0)$, along the n axis is presented in this figure. These results (curve A of fig. 5) are presented in a log-log plot to show the slow convergence at $z = 0$. When curves B and C of figure 5 are plotted in the same manner, they fall off to zero much more rapidly.

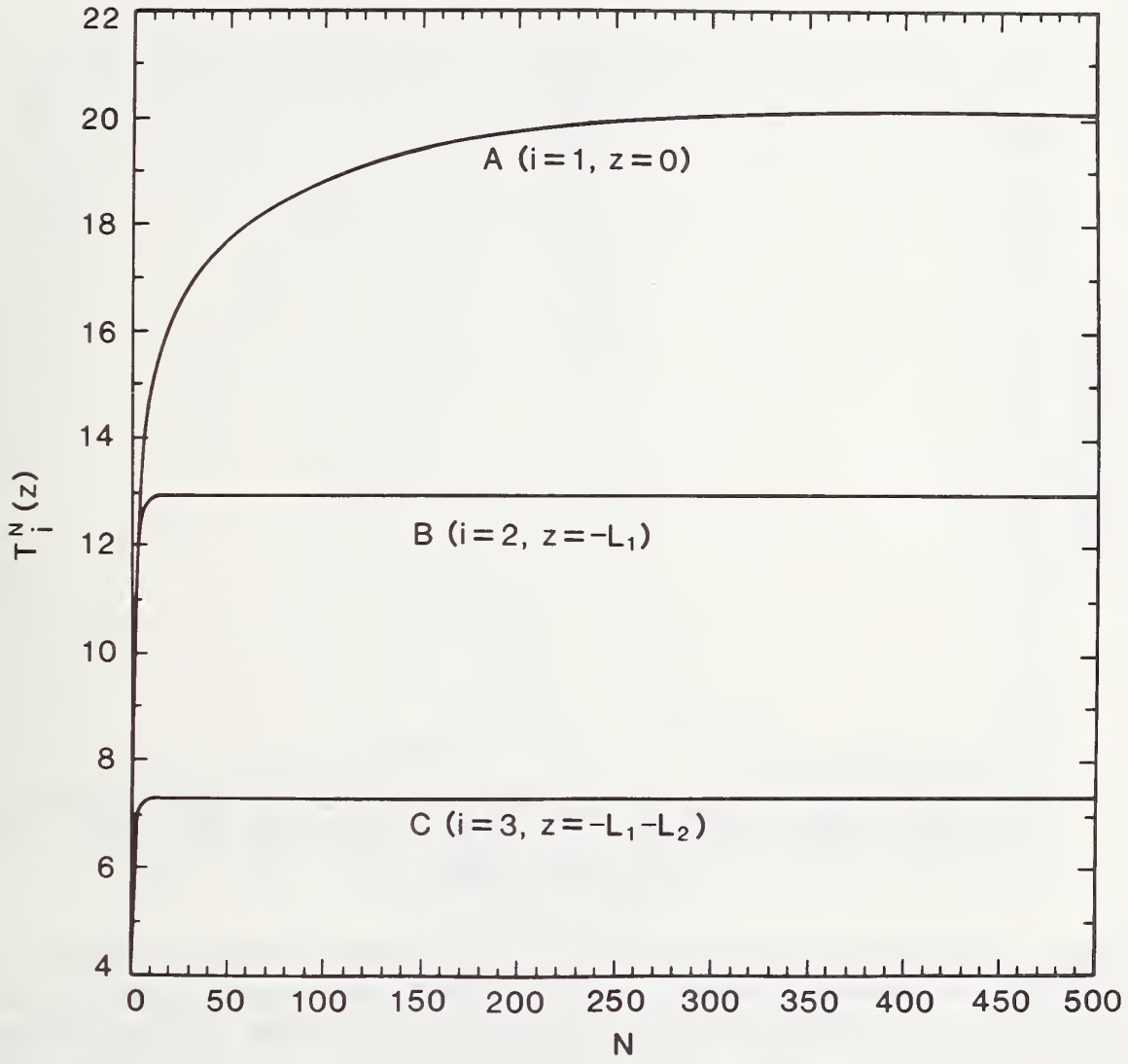

Figure 7. This figure shows the calculated temperature as a function of the number of terms used in the sum (eq (98)) for the top of each of the three layers. Curve A is $T_1^N(z=0)$ while curves B and C represent $T_2^N(z=-L_1)$ and $T_3^N(z=-L_1-L_2)$, respectively. This figure clearly shows how the behavior of the Fourier coefficients for each of the layers is mirrored in the corresponding calculated temperatures.

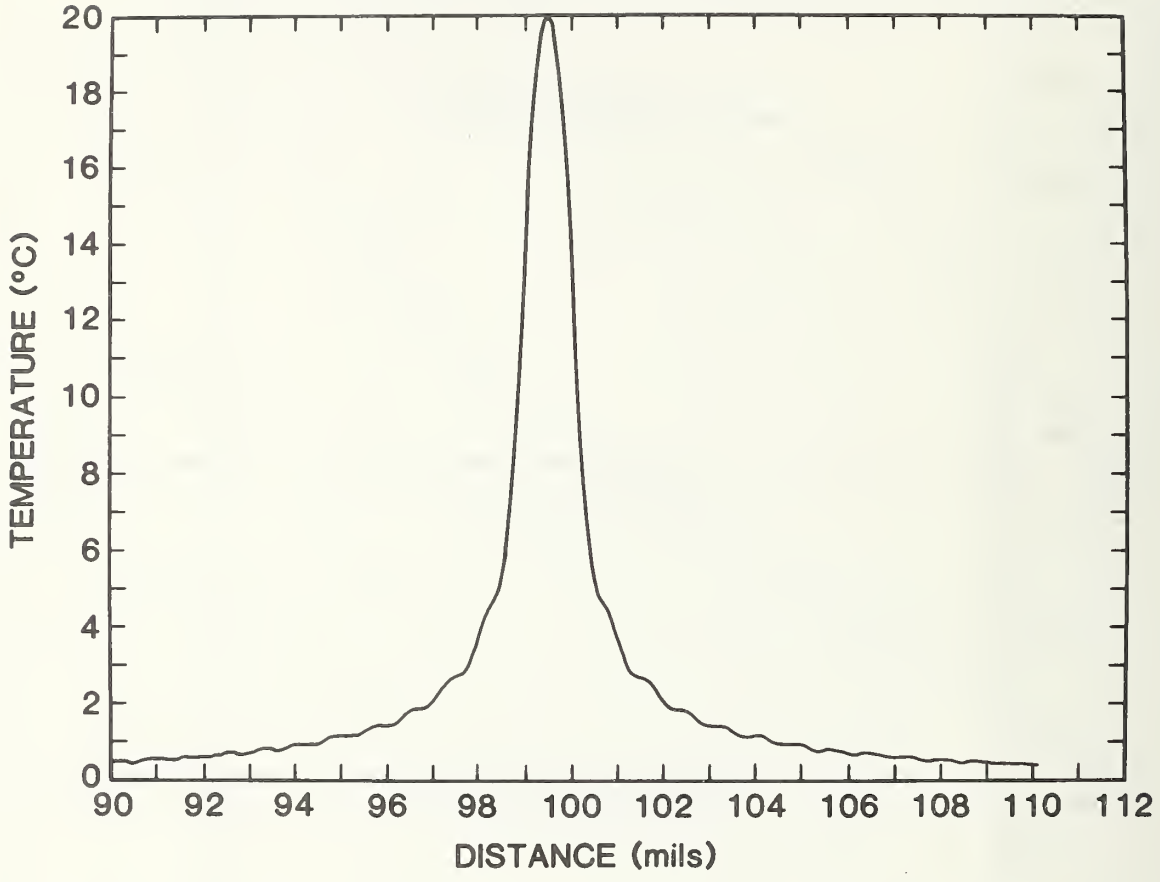

Figure 8. This figure presents the calculated temperature along the midline of a 200 mil by 200 mil rectangular structure with a single, uniform power density 1 mil by 1 mil heat source located at the center of the surface of the top layer. Note that only the temperature in the region of the heat source is plotted, i.e., from 90 mils to 110 mils. Also, note that the temperature falls off rapidly away from the heat source. The specific structure is that of 15 mils of silicon on 2 mils of die attach material on 30 mils of substrate material. The annotated input data file used to obtain this temperature data is to be found in Table I.

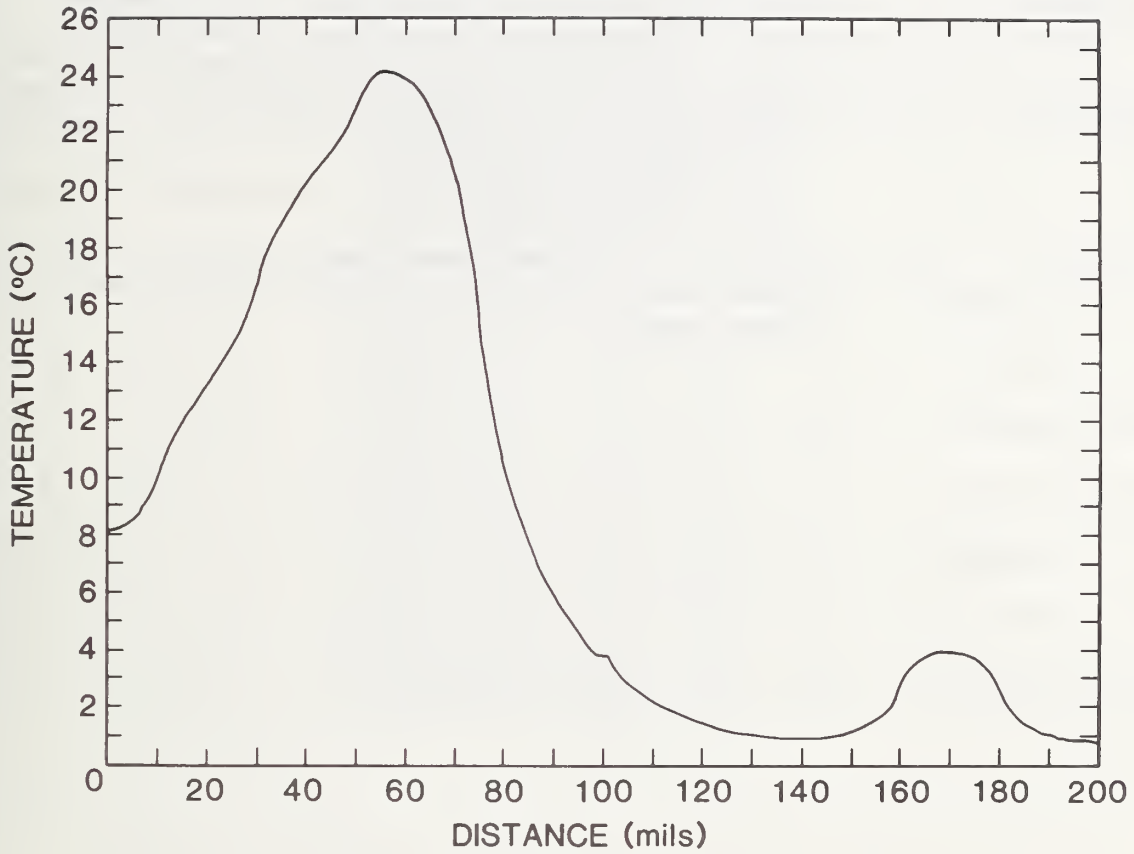

Figure 9. By way of contrast with the previous figure, the calculated temperature along the midline of a 200 mil by 200 mil three-layer structure is presented in this figure. However, in this case, two nonuniform heat sources and one uniform heat source are used. The three-layer structure is the same as used in the previous figure. The construction of the nonuniform heat sources from a number of uniform heat sources is contained in the annotated input data file contained in Table II.

TABLE I
INPUT DATA FOR SINGLE HEAT SOURCE

INPUT DATA	DESCRIPTION OF INPUT DATA
200 150	X AND Y DIMENSIONS OF RECTANGULAR STRUCTURE
15 0.00267	THICKNESS AND THERMAL CONDUCTIVITY OF TOP LAYER
2 0.00064	THICKNESS AND THERMAL CONDUCTIVITY OF MIDDLE LAYER
30 0.00999	THICKNESS AND THERMAL CONDUCTIVITY OF BOTTOM LAYER
500 500	UPPER SUMMATION LIMITS FOR N AND M SUMMATIONS
1	IEDGEX (=0 FOR SINGLE POINT, =1 FOR NUMBER OF POINTS)
201 90 0.1	NUMBER OF POINTS ALONG X, FIRST POINT, STEP INCREMENT
0	IEDGEY (=0 FOR SINGLE POINT, =1 FOR NUMBER OF POINTS)
75.5	Y POINT FOR CALCULATION
0	IEDGEZ (=0 FOR SINGLE POINT, =1 FOR NUMBER OF POINTS)
0	Z POINT FOR CALCULATION
1 0.093423	NUMBER OF HEAT SOURCES AND POWER DENSITY
99 1 75 1	X COORDINATE, LENGTH ALONG X AXIS, Y COORDINATE, LENGTH ALONG Y AXIS
0	
0	
0 0	
0	

TABLE II
INPUT DATA FOR MULTIPLE, NONUNIFORM HEAT SOURCES

INPUT DATA	DESCRIPTION OF INPUT DATA
200 200	X AND Y DIMENSIONS OF RECTANGULAR STRUCTURE
15 0.00267	THICKNESS AND THERMAL CONDUCTIVITY OF TOP LAYER
2 0.00064	THICKNESS AND THERMAL CONDUCTIVITY OF MIDDLE LAYER
30 0.00999	THICKNESS AND THERMAL CONDUCTIVITY OF BOTTOM LAYER
500 500	UPPER SUMMATION LIMITS FOR N AND M SUMMATIONS
1	IEDGEX (=0 FOR SINGLE POINT, =1 FOR NUMBER OF POINTS)
200 0 1	NUMBER OF POINTS ALONG X, FIRST POINT, STEP INCREMENT
0	IEDGEY (=0 FOR SINGLE POINT, =1 FOR NUMBER OF POINTS)
101	Y POINT FOR CALCULATION
0	IEDGEZ (=0 FOR SINGLE POINT, =1 FOR NUMBER OF POINTS)
0	Z POINT FOR CALCULATION
7 0.001	NUMBER OF HEAT SOURCES AND POWER DENSITY
10 65 10 180	X COORDINATE, LENGTH ALONG X AXIS, Y COORDINATE, LENGTH ALONG Y AXIS FOR HEATER #1
30 45 10 180	X COORDINATE, LENGTH ALONG X AXIS, Y COORDINATE, LENGTH ALONG Y AXIS FOR HEATER #2
50 25 10 180	X COORDINATE, LENGTH ALONG X AXIS, Y COORDINATE, LENGTH ALONG Y AXIS FOR HEATER #3
100 2 100 2	X COORDINATE, LENGTH ALONG X AXIS, Y COORDINATE, LENGTH ALONG Y AXIS FOR HEATER #4
160 20 10 20	X COORDINATE, LENGTH ALONG X AXIS, Y COORDINATE, LENGTH ALONG Y AXIS FOR HEATER #5
160 20 90 20	X COORDINATE, LENGTH ALONG X AXIS, Y COORDINATE, LENGTH ALONG Y AXIS FOR HEATER #6
160 20 170 20	X COORDINATE, LENGTH ALONG X AXIS, Y COORDINATE, LENGTH ALONG Y AXIS FOR HEATER #7
0	
0	
0 0	
0	

APPENDIX A--TXYZ PROGRAM LISTING

MAIN PROGRAM LISTING

```
C     ****************************************************************
C     INSTRUCTIONS FOR INPUT DATA FILE SETUP
C     DATA TO BE READ BY THIS PROGRAM ARE SET UP AS FOLLOWS:
C
C     RLX (X DIMENSION OF 3 LAYER STRUCTURE)
C     RLY (Y DIMENSION OF 3 LAYER STRUCTURE)
C     RLS (THICKNESS OF TOP LAYER)
C     RKS (THERMAL CONDUCTIVITY OF TOP LAYER)
C     RLC (THICKNESS OF MIDDLE LAYER)
C     RKC (THERMAL CONDUCTIVITY OF MIDDLE LAYER)
C     RLI (THICKNESS OF BOTTOM LAYER)
C     RKI (THERMAL CONDUCTIVITY OF BOTTOM LAYER)
C     NUP (UPPER LIMIT OF N SUM, X DIRECTION)
C     MUP (UPPER LIMIT OF M SUM, Y DIRECTION)
C
C     IEDGEX (=0 FOR DEFAULT, =1 TO ALTER RANGE OF X VALUES)
C     IF IEDGEX=1 THEN THE FOLLOWING SEGMENT OF INPUT IS READ
C                 ILX (THE NUMBER OF INCREMENTS IN X TO BE USED)
C                     (ILX+1=THE NUMBER OF X VALUES TO BE USED)
C                 X1 (THE VALUE OF THE FIRST POINT IN X)
C                 STEPX (THE INCREMENT IN X)
C     ELSE IEDGEX=0  READ X1 (THE CONSTANT X VALUE)
C
C     IEFGEY (=0 FOR DEFAULT, =1 TO ALTER RANGE OF Y VALUES)
C     IF IEDGEY=1 THEN THE FOLLOWING SEGMENT OF INPUT IS READ
C                 ILY (THE NUMBER OF INCREMENTS IN Y TO BE USED)
C                     (ILY+1=THE NUMBER OF Y VALUES TO BE USED)
C                 Y1 (THE VALUE OF THE FIRST POINT IN Y)
C                 STEPY (THE INCREMENT IN Y)
C     ELSE IEDGEX=0  READ X1 (THE CONSTANT X VALUE)
C
C     IEDGEZ (=0 FOR DEFAULT, =1 TO ALTER RANGE OF Z VALUES)
C     IF IEDGEZ=1 THEN THE FOLLOWING SEGMENT OF INPUT IS READ
C                 ILZ (THEN NUMBER OF INCREMENTS IN Z TO BE USED)
C                     (ILZ+1=THE NUMBER OF Z VALUES TO BE USED)
C                 Z1 (THE VALUE OF THE FIRST POINT IN Z)
C                 STEPZ (THE INCREMENT IN Z)
C     ELSE IEDGEX=0  READ X1 (THE CONSTANT X VALUE)
C
C     NOTE: AS THE CALCULATION TAKES THE Z VARIABLE TO BE ZERO OR
C           NEGATIVE, ENTER Z1 AND STEPZ AS POSITIVE QUANTITIES.
C           THE PROGRAM CHANGES THE SIGN OF Z1 AND STEPZ TO MAKE
C           THE CALCULATION FOR NEGATIVE Z.
C
C     NSOUR (NUMBER OF HEAT SOURCES, UP TO 20)
C     PO (POWER DENSITY)
C     XSOUR(1) (X COORDINATE OF ORIGIN OF 1ST SOURCE)
```

```
C     YSOUR(1) (Y COORDINATE OF ORIGIN OF 1ST SOURCE)
C     RLXSOUR(1) (LENGTH ALONG X DIRECTION OF 1ST SOURCE)
C     RLYSOUR(1) (LENGTH ALONG Y DIRECTION OF 1ST SOURCE)
C     REMAINING HEAT SOURCES WITH SAME INPUT STRUCTURE AS
C
C     ****************************************************************
C
      DIMENSION X(501),Y(501),Z(501),COSYT(501)
      DIMENSION ARUZER(501,501),ARFUNZ(501,501)
      DIMENSION XSOUR(20),YSOUR(20),RLXSOR(20),RLYSOR(20)
      COMMON RKS,RKC,RKI,RLX,RLY,RLS,RLC,RLI
      COMMON NSOUR,XSOUR,YSOUR,RLXSOR,RLYSOR
C
C     ************************************************************
C
C     DOCUMENTATION AND BACKGROUND
C     THIS PROGRAM CALCULATES THE SURFACE TEMPERATURE T(X,Y,Z) DUE TO DC
C     POWER INPUTS ONLY.  THE TEMPERATURE IS THE TEMPERATURE RELATIVE
C     TO THE AMBIENT.  THE SPECIFIC EQUATIONS USED ARE GIVEN IN
C     EQUATIONS (13)-(23), WITH S=0 (STEADY-STATE CONDITION),
C     IN THE PAPER BY KOKKAS (REF: "THERMAL ANALYSIS OF MULTIPLE-
C     LAYERED STRUCTURES" BY ACHILLES G. KOKKAS, IEEE TRANS. ELEC. DEV.
C     VOL. ED-21, NO. 11, 674-681 (1974)).
C     VARIABLES USED: THE VARIABLES LISTED AS REAL IN THE ABOVE
C     ARE THE FOLLOWING-KS,KC, AND KI ARE THE THERMAL CONDUCTIVITIES
C     OF THE SEMICONDUCTOR, CONDUCTOR, AND INSULATOR, RESPECTIVELY.
C     LX AND LY ARE THE LATERAL DIMENSION OF THE CHIP WHILE
C     LS,LC, AND LI ARE THE THICKNESSES OF THE SEMICONDUCTOR, THE
C     CONDUCTOR, AND THE INSULATOR, RESPECTIVELY.
C     IMPORTANT NOTE:  WHILE THE VARIABLES HAVE THE NOTATION WHICH
C     SEEMS TO IMPLY A SEMICONDUCTOR, A CONDUCTOR, AND AN INSULATOR,
C     THESE REFER TO THE WAY IN WHICH KOKKAS FORMULATED THE PROBLEM.
C     THE THING TO KEEP IN MIND IS THAT THE TOP LAYER HAS (LS,KS),
C     THE MIDDLE LAYER HAS (LC,KC), AND THE BOTTOM LAYER HAS (LI,KI).
C     IT IS NOT NECESSARY THAT THEY BE WHAT THEY SEEM TO BE, THEY ARE
C     DETERMINED BY THE RESPECTIVE THICKNESSES, L, AND CONDUCTIVITIES,K.
C
C     ************************************************************
C
 1    FORMAT(1H1)
 2    FORMAT(1X,'STEADY-STATE THERMAL ANALYSIS CALCULATION USING EQS.
     1 (13)-(23) OF KOKKAS'/)
 3    FORMAT(1X,'THERMAL CONDUCTIVITIES AND LAYER THICKNESSES')
 4    FORMAT(1X,'KS= ',F10.8,' KC= ',F10.8,' KI= ',F10.8)
 5    FORMAT(1X,'LS= ',F10.5,' LC= ',F10.5,' LI= ',F10.5)
 6    FORMAT(/1X,'UPPER SUMMATION LIMITS ',2X,' NUP=',I5,
     1 '  MUP=',I5/)
 7    FORMAT(//1X,'NUMBER OF HEAT SOURCES=',I5)
 8    FORMAT(/1X,'COORDINATES, LENGTHS, AND WIDTHS OF HEAT SOURCES'/)
 9    FORMAT(1X,'HEAT SOURCE ',7X,'XSOUR',10X,'YSOUR',9X,'LXSOUR',
     1 9X,'LYSOUR'/)
 10   FORMAT(7X,I3,5X,F10.5,5X,F10.5,5X,F10.5,5X,F10.5)
```

```
 11   FORMAT(1X,'POWER DENSITY=',F12.6)
 12   FORMAT(/1X,'CALCULATING ',I3,' X POINTS WITH A FIRST POINT OF '
     1  ,F5.1,' AND A STEP SIZE OF ',F5.1)
 13   FORMAT(/1X,'THE CONSTANT X COORDINATE IS ',F5.1)
 14   FORMAT(/1X,'CALCULATING ',I3,' Y POINTS WITH A FIRST POINT OF '
     1  ,F5.1,' AND A STEP SIZE OF ',F5.1)
 15   FORMAT(/1X,'THE CONSTANT Y COORDINATE IS ',F5.1)
 16   FORMAT(/1X,'CALCULATING ',I3,' Z POINTS WITH A FIRST POINT OF '
     1  ,F5.1,' AND A STEP SIZE OF ',F5.1)
 17   FORMAT(/1X,'THE CONSTANT Z COORDINATE IS ',F5.1)
 27   FORMAT(1X,'LX= ',F7.2,3X,' LY= ',F7.2)
 22   FORMAT(1X,3F8.2,F10.4)
 31   FORMAT(1X,6I7)
 51   FORMAT(I4,3X,I4)
 52   FORMAT(F10.5,3X,F10.5)
 53   FORMAT(I1)
 54   FORMAT(I4,3X,F10.5,3X,F10.5)
 55   FORMAT(F10.5)
 56   FORMAT(I2,3X,F10.5)
 57   FORMAT(F10.5,3X,F10.5,3X,F10.5,3X,F10.5)
C
C     ****************************************************************
C     DATA INPUT SECTION (DATA READ FROM FOR010)
C     ****************************************************************
C
      IXFLG = 0
      IYFLG = 0
      IZFLG = 0
      ILX=0
      ILY=0
      ILZ=0
      STEPX=1.0
      STEPY=1.0
      STEPZ=-1.0
      WRITE(11,1)
      WRITE(11,2)
      WRITE(11,3)
C     RLX AND RLY ARE THE X AND Y DIMENSIONS OF THE RECTANGULAR STRUCTURE
      READ(10,52)RLX,RLY
      WRITE(11,27)RLX,RLY
C     RLS IS THE THICKNESS AND RKS IS THE THERMAL CONDUCTIVITY OF THE TOP LAYER
      READ(10,52)RLS,RKS
C     RLC IS THE THICKNESS AND RKC IS THE THERMAL CONDUCTIVITY THE MIDDLE LAYER
      READ(10,52)RLC,RKC
C     RLI IS THE THICKNESS AND RKI IS THE THERMAL CONDUCTIVITY THE BOTTOM LAYER
      READ(10,52)RLI,RKI
      WRITE(11,5)RLS,RLC,RLI
      WRITE(11,4)RKS,RKC,RKI
C     NUP IS THE UPPER LIMIT OF THE SUMMATION OVER THE INDEX N (X-DIR)
C     MUP IS THE UPPER LIMIT OF THE SUMMATION OVER THE INDEX M (Y-DIR)
      READ(10,51)NUP,MUP
      WRITE(11,6)NUP,MUP
```

```
      NUP = NUP + 1
      MUP = MUP + 1
      IF (NUP.GT.501.OR.MUP.GT.501) GO TO 3999
 888  FORMAT(1X,'YOUR UPPER LIMIT OF SUMMATION IS TOO LARGE. TRY AGAIN')
      READ(10,53)IEDGEX
C     SET IEDGEX=0 TO GET CONSTANT POINT FOR X
C     SET IEDGEX=1 TO SET THE LIMITS OF THE CALCULATION ALONG X
      IF(IEDGEX.EQ.0) GO TO 115
      READ(10,54)ILX,X1,STEPX
      WRITE(11,12)ILX,X1,STEPX
      GO TO 125
 115  READ(10,55)X1
      WRITE(11,13)X1
      IXFLG = 1
 125  READ(10,53)IEDGEY
C     SET IEDGEY=0 TO GET CONSTANT POINT FOR Y
C     SET IEDGEY=1 TO SET THE LIMITS OF THE CALCULATION ALONG Y
      IF(IEDGEY.EQ.0) GO TO 135
      READ(10,54)ILY,Y1,STEPY
      WRITE(11,14)ILY,Y1,STEPY
      GO TO 150
 135  READ(10,55)Y1
      WRITE(11,15)Y1
      IYFLG = 1
 150  READ(10,53)IEDGEZ
C     SET IEDGEZ=0 TO GET CONSTANT POINT FOR Z
C     SET IEDGEZ=1 TO SET THE LIMITS OF THE CALCULATION ALONG Z
      IF (IEDGEZ.EQ.0) GO TO 160
      READ (10,54)ILZ,Z1,STEPZ
      WRITE(11,16)ILZ,Z1,STEPZ
      Z1=-1.0*Z1
      STEPZ=-1.0*STEPZ
      GOTO 175
 160  READ(10,55)Z1
      WRITE(11,17)Z1
      Z1 = -1.0*Z1
      IZFLG = 1
      ILX = ILX + 1
      ILY = ILY + 1
      ILZ = ILZ + 1
 175  READ(10,56)NSOUR,PO
C     PO IS THE POWER DENSITY, ASSUMED UNIFORM FOR ALL HEATERS
C     NSOUR IS THE TOTAL NUMBER OF HEATING ELEMENTS ON THE SURFACE OF THE
C     THE TOP LAYER (UP TO 20 ARE POSSIBLE)
      WRITE(11,7)NSOUR
      WRITE(11,11) PO
      WRITE(11,8)
      WRITE(11,9)
C     THE NEXT LOOP READS IN THE COORDINATES OF THE ORIGIN OF THE
C     HEATING ELEMENTS ALONG WITH THEIR LENGTHS AND WIDTHS
      DO 100 I=1,NSOUR
      READ(10,57)XSOUR(I),RLXSOR(I),YSOUR(I),RLYSOR(I)
```

```
C     XSOUR(I) IS THE X COORDINATE OF THE ORIGIN OF I-TH HEATER ELEMENT
C     RLXSOR(I) IS THE LENGTH OF THE I-TH HEATER ALONG THE X DIRECTION
C     YSOUR(I) IS THE Y COORDINATE OF THE ORIGIN OF I-TH HEATER ELEMENT
C     RLYSOR(I) IS THE LENGTH OF THE I-TH HEATER ALONG THE Y DIRECTION
      WRITE(11,10)I,XSOUR(I),YSOUR(I),RLXSOR(I),RLYSOR(I)
 100  CONTINUE
      WRITE(11,1)
C
C     ****************************************************************
C
C     END OF DATA INPUT SECTION
C     END OF INPUT SECTION.  THE THERMAL CONDUCTIVITIES OF THE
C     SEMICONDUCTOR, CONDUCTOR, AND INSULATOR, THE THICKNESSES OF THE
C     SEMICONDUCTOR, CONDUCTOR, AND INSULATOR, THE X AND Y DIMENSIONS
C     OF THE CHIP, THE NUMBER OF HEATING SOURCES AND THEIR X,Y AND
C     LENGTH AND WIDTH HAVE BEEN ENTERED.
C
C     ****************************************************************
C
  180 PI=3.14159265
      P04LK = 4.0 * P0 / (RLX*RLY*RKS)
      PILX = PI / RLX
      PILY = PI / RLY
C
C     ****************************************************************
C     CALCULATE THE X(0:INTX), Y(0:INTY), AND Z(0:INTZ) ARRAYS
C     ****************************************************************
C
      X(1)=X1
      DO 200 I=2,ILX
      X(I)=X(I-1)+STEPX
 200  CONTINUE
      Y(1)=Y1
      DO 220 I=2,ILY
      Y(I)=Y(I-1)+STEPY
 220  CONTINUE
      Z(1)=Z1
      DO 240 I=2,ILZ
      Z(I)=Z(I-1)+STEPZ
 240  CONTINUE
C
C     ****************************************************************
C
C     BEGIN CALCULATION OF T(X,Y,Z)
C     THE SUBROUTINES USED IN THE CALCULATION ARE:
C     1) UZERO(N,M) - CALCULATES THE FOURIER COSINE TRANSFORM OF THE
C        FUNCTION, U(X,Y), THE POWER DENSITY FUNCTION FOR ALL OF THE
C        HEAT SOURCES.
C     2) FUNZ(N,M,Z) - CALCULATES THE Z-DEPENDENT PORTION OF THE SUM
C        REMEMBERING THAT THIS IS A FUNCTION OF THE SUMMATION
C        INDICES (N,M).
C
```

```
C     ****************************************************************
C
C
C     ****************************************************************
C     CALCULATE THE FOURIER COMPONENTS OF THE HEAT SOURCES, U(N,M)
C     ****************************************************************
C
      DO 300 MM=1,MUP
      M = MM - 1
      DO 250 NN=1,NUP
         N = NN - 1
         ARUZER(NN,MM)=UZERO(N,M)
 250  CONTINUE
 300  CONTINUE
C
C     ****************************************************************
C
C     END OF U(N,M) CALCULATION AND BEGINNING OF MAJOR LOOP FOR Z
C
C     ****************************************************************
C
      DO 3000 IZ=1,ILZ
C
C     ****************************************************************
C     CALCULATE THE Z DEPENDENT POTION, I.E., FUNZ(N,M,Z)
C     ****************************************************************
C
      DO 400 MM=1,MUP
      M = MM - 1
      DO 350 NN=1,NUP
         N = NN - 1
         ARFUNZ(NN,MM)=FUNZ(N,M,Z(IZ))*ARUZER(NN,MM)
 350  CONTINUE
 400  CONTINUE
C
      DO 3000  IY=1,ILY
      DO 700 MM=1,MUP
         M = MM - 1
         COSYT(MM)=COS(FLOAT(M)*Y(IY)*PILY)
 700  CONTINUE
C
      DO 3000 IX=1,ILX
         SUM=0.0
         DO 1900 MM=1,MUP
         M = MM - 1
         DO 1700 NN=1,NUP
            N = NN - 1
            NDN=0
            NDM=0
            IF (N.EQ.0) NDN=1
            IF (M.EQ.0) NDM=1
            TOP = ARFUNZ(NN,MM) * COS(FLOAT(N)*X(IX)*PILX) * COSYT(MM)
```

```
            BOTTOM=(NDN+1)*(NDM+1)
            TSUM=TOP/BOTTOM
            SUM=SUM+TSUM
 1700     CONTINUE
 1900     CONTINUE
          TEMP = P04LK * SUM
          WRITE (20,22)X(IX),Y(IY),Z(IZ),TEMP
 3000 CONTINUE
      GO TO 4000
 3999 WRITE(6,888)
 4000 STOP
      END
C     ****************************************************************
C     END OF THE MAIN PROGRAM
C     ****************************************************************
```

FUNCTION UZERO LISTING

```
      FUNCTION UZERO(N,M)
      DIMENSION XSOUR(20),YSOUR(20),RLXSOR(20),RLYSOR(20)
      COMMON RKS,RKC,RKI,RLX,RLY,RLS,RLC,RLI
      COMMON NSOUR,XSOUR,YSOUR,RLXSOR,RLYSOR
C
C     ****************************************************************
C     DESCRIPTION OF THE FUNCTION UZERO(N,M)
C     THIS FUNCTION CALCULATES THE DOUBLE FOURIER COSINE TRANSFORM
C     OF THE POWER DENSITY FUNCTION, U(X,Y).  THIS IS THE TRANSFORM
C     FOR ALL OF THE HEAT SOURCES.  THE ASSUMPTION IS MADE THAT THE
C     POWER DENSITY IS UNIFORM AND EQUAL TO UNITY OVER THE SURFACE
C     OF THE HEATING ELEMENTS.  THAT IS,
C       U(X,Y)=1  (XSOUR(I)<=X<=XSOUR(I)+LXSOUR(I)  AND
C                  YSOUR(I)<=Y<=YXOUR(I)+LYSOUR(I) ).
C       U(X,Y)=0  OTHERWISE.
C     UNDER THESE CONDITIONS, IT IS POSSIBLE TO ANALYTICALLY EVALUATE
C     THE DOUBLE INTEGRAL FOR EACH HEATING ELEMENT.  AS THE HEATING
C     ELEMENTS ARE ASSUMED TO BE INDEPENDENT, THE CONTRIBUTION FROM
C     EACH ELEMENT MAY BE ADDED TO OBTAIN THE U(N,M) FOR ALL.
C
C     ****************************************************************
C
      PI=3.14159265
      UZERO=0.0
      DO 500 I=1,NSOUR
         IF(N.EQ.0) GO TO 100
           TERMX = SIN(FLOAT(N)*PI*(XSOUR(I) + RLXSOR(I))/RLX)
     1             - SIN(FLOAT(N)*PI*XSOUR(I)/RLX)
           TERMX=TERMX*RLX/(FLOAT(N)*PI)
           GO TO 150
  100    TERMX=RLXSOR(I)
  150    IF(M.EQ.0) GO TO 200
           TERMY = SIN(FLOAT(M)*PI*(YSOUR(I) + RLYSOR(I))/RLY)
     1             - SIN(FLOAT(M)*PI*YSOUR(I)/RLY)
           TERMY=TERMY*RLY/(FLOAT(M)*PI)
           GO TO 250
  200    TERMY=RLYSOR(I)
  250    TERMI=TERMX*TERMY
         UZERO=UZERO+TERMI
  500 CONTINUE
      RETURN
      END
C     ****************************************************************
```

APPENDIX A--TXYZ PROGRAM LISTING

FUNCTION FUNZ LISTING

```
      FUNCTION FUNZ(N,M,Z)
      DIMENSION XSOUR(20),YSOUR(20),RLXSOR(20),RLYSOR(20)
      COMMON RKS,RKC,RKI,RLX,RLY,RLS,RLC,RLI
      COMMON NSOUR,XSOUR,YSOUR,RLXSOR,RLYSOR
C
C     ****************************************************************
C
C     DESCRIPTION OF THE FUNCTION FUNZ(N,M,Z)
C     THIS FUNCTION IS USED TO CALCULATE THE Z DEPENDENT PART OF THE
C     FUNCTION USING THE S=0 VERSIONS OF EQUATIONS (15)-(17) OF KOKKAS.
C     THESE ARE USED IN CONJUNCTION WITH EQUATIONS (18)-(22) ALSO FOR
C     S=0 (STEADY-STATE CONDITION).  THE SPECIFIC FORM OF FUNZ IS
C     DETERMINED BY THE VALUE OF Z, I.E., IF Z FALLS IN THE TOP, MIDDLE
C     OR BOTTOM LAYER OF THE STRUCTURE.  IN ADDITION, SPECIAL CARE IS
C     TAKEN AS TO EVALUATE FUNZ FOR THE CASE WHERE GAMMA=0 AS THESE ARE
C     SIMPLE, BUT THE COMPUTER DOES NOT KNOW HOW TO EVALUATE LIMITS.
C     IN ADDITION, THE FORM OF THE THREE SOLUTIONS HAS BEEN CHANGED TO
C     GET RID OF THE ARTIFICIAL OVERFLOW PROBLEMS COMING FROM THE
C     HYPERBOLIC FUNCTIONS, COSH AND SINH, FOR THE CASES WHERE THE
C     ARGUMENTS BECOME LARGE.
C
C     ****************************************************************
C
      PI=3.14159265
      GAMMA=SQRT((FLOAT(N)*PI/RLX)**2 + (FLOAT(M)*PI/RLY)**2)
      VS=GAMMA*RLS
      VC=GAMMA*RLC
      VI=GAMMA*RLI
      VT=GAMMA*(RLS+Z)
      VM=GAMMA*(RLS+RLC+Z)
      VB=GAMMA*(RLS+RLC+RLI+Z)
      BOT1=TANH(VS)*TANH(VI)
      BOT1=BOT1+(RKI/RKC)*TANH(VS)*TANH(VC)
      BOT2=(RKC/RKS)*TANH(VI)*TANH(VC)+(RKI/RKS)
      GFUNC=1.0/(BOT1+BOT2)
      AZ=ABS(Z)
      IF (AZ.GT.RLS) GO TO 500
```

```
C     ****************************************************************
C            TOP LAYER CALCULATION
C     THIS PORTION IS THE TOP LAYER CALCULATION WHICH IS DEFAULTED
C     TO IF Z FALLS INTO THE TOP LAYER
C
      IF (GAMMA.EQ.0.0) GO TO 100
      TERMS1=TANH(VI)+(RKI/RKC)*TANH(VC)
      TERMS2=(RKC/RKS)*TANH(VT)*(TANH(VI)*TANH(VC)+(RKI/RKC))
      TERMS=TERMS1+TERMS2
      IF (Z.EQ.0.0) GO TO 90
      IF (VS.GT.5.0.AND.VT.GT.5.0) GO TO 80
      IF (VS.LT.5.0) GO TO 10
      C1=2.0*EXP(-VS)
      GO TO 20
 10   C1=1.0/COSH(VS)
 20   CONTINUE
      IF (VT.LT.5.0) GO TO 30
      C2=0.5*EXP(VT)
      GO TO 40
 30   C2=COSH(VT)
 40   CONTINUE
      FUNZ=GFUNC*TERMS*C1*C2/GAMMA
      RETURN
 80   FUNZ=GFUNC*TERMS*EXP(GAMMA*Z)/GAMMA
      RETURN
 90   FUNZ=GFUNC*TERMS/GAMMA
      RETURN
 100  FUNZ=(RLS+Z)+(RKS/RKC)*RLC+(RKS/RKI)*RLI
      RETURN
 500  TOTAL=RLS+RLC
      IF (AZ.GT.TOTAL) GO TO 1500
```

```
C     ****************************************************************
C            MIDDLE LAYER CALCULATION
C     THIS IS THE MIDDLE LAYER CALCULATION WHICH IS DEFAULTED TO IF
C     Z FALLS INTO THIS DOMAIN OF DEPTHS
C
      IF (GAMMA.EQ.0.0) GO TO 1000
      TERMC=TANH(VI)+(RKI/RKC)*TANH(VM)
      IF (VS.GT.5.0.AND.VC.GT.5.0.AND.VM.GT.5.0) GO TO 800
      IF (VS.LT.5.0) GO TO 250
      C1=2.0*EXP(-VS)
      GO TO 260
  250 C1=1.0/COSH(VS)
  260 CONTINUE
      IF (VC.LT.5.0) GO TO 270
      C2=2.0*EXP(-VC)
      GO TO 280
  270 C2=1.0/COSH(VC)
  280 CONTINUE
      IF (VM.LT.5.0) GO TO 320
      C3=0.5*EXP(VM)
      GO TO 330
  320 C3=COSH(VM)
  330 CONTINUE
      FUNZ=GFUNC*TERMC*C1*C2*C3/GAMMA
      RETURN
  800 FUNZ=GFUNC*TERMC*2.0*EXP(GAMMA*Z)/GAMMA
      RETURN
 1000 FUNZ=(RKS/RKI)*RLI+(RKS/RKC)*(RLS+RLC+Z)
      RETURN
```

```
C     ****************************************************************
C           BOTTOM LAYER CALCULATION
C     THIS IS THE BOTTOM LAYER CALCULATION WHICH IS USED IF Z FALLS
C     INTO THE BOTTOM LAYER
C
 1500 IF (GAMMA.EQ.0.0) GO TO 2000
      IF (VS.GT.5.0.AND.VC.GT.5.0.AND.VI.GT.5.0.AND.
     1 VB.GT.5.0) GO TO 1900
      IF(VB.GT.5.0.AND.VS.GT.5.0.AND.VC.LT.5.0.AND.VI.LT.5.0)GO TO 2100
      IF(VB.GT.5.0.AND.VS.LT.5.0.AND.VC.GT.5.0.AND.VI.LT.5.0)GO TO 2200
      IF(VB.GT.5.0.AND.VS.LT.5.0.AND.VC.LT.5.0.AND.VI.GT.5.0)GO TO 2300
      IF(VB.GT.5.0.AND.VS.GT.5.0.AND.VC.GT.5.0.AND.VI.LT.5.0)GO TO 2400
      IF(VB.GT.5.0.AND.VS.GT.5.0.AND.VC.LT.5.0.AND.VI.GT.5.0)GO TO 2500
      IF(VB.GT.5.0.AND.VS.LT.5.0.AND.VC.GT.5.0.AND.VI.GT.5.0)GO TO 2600
      IF (VS.LT.5.0) GO TO 1550
      C1=2.0*EXP(-VS)
      GO TO 1560
 1550 C1=1.0/COSH(VS)
 1560 CONTINUE
      IF (VC.LT.5.0) GO TO 1570
      C2=2.0*EXP(-VC)
      GO TO 1580
 1570 C2=1.0/COSH(VC)
 1580 CONTINUE
      IF (VI.LT.5.0) GO TO 1590
      C3=2.0*EXP(-VI)
      GO TO 1600
 1590 C3=1.0/COSH(VI)
 1600 CONTINUE
      C4=SINH(VB)
      FUNZ=GFUNC*C1*C2*C3*C4/GAMMA
      RETURN
 1900 FUNZ=GFUNC*4.0*EXP(GAMMA*Z)/GAMMA
      RETURN
 2000 FUNZ=(RKS/RKI)*(RLS+RLC+RLI+Z)
      RETURN
 2100 FUNZ=GFUNC*EXP(VB-VS)/(GAMMA*COSH(VC)*COSH(VI))
      RETURN
 2200 FUNZ=GFUNC*EXP(VB-VC)/(GAMMA*COSH(VS)*COSH(VI))
      RETURN
 2300 FUNZ=GFUNC*EXP(VB-VI)/(GAMMA*COSH(VS)*COSH(VC))
      RETURN
 2400 FUNZ=GFUNC*2.0*EXP(VB-VS-VC)/(GAMMA*COSH(VI))
      RETURN
 2500 FUNZ=GFUNC*2.0*EXP(VB-VS-VI)/(GAMMA*COSH(VC))
      RETURN
 2600 FUNZ=GFUNC*2.0*EXP(VB-VC-VI)/(GAMMA*COSH(VS))
      RETURN
      END
```

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